FIITJEE

ALL INDIA TEST SERIES

FULL TEST – IX

JEE (Advanced)-2021

PAPER –1

TEST DATE: 11-09-2021

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A

Sol. For plank

$f + 800 = 900$

$f = 100$

for man

$300 + f = 50a$

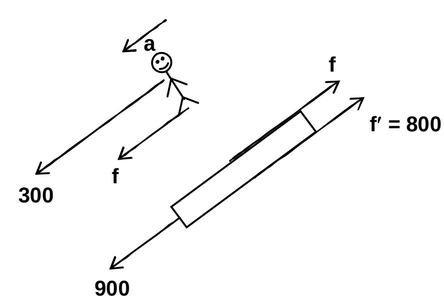
$400 = 50a$

$a = 8\ m/s^2$

$S = ut + \frac{1}{2}at^2$

$16 = 0 + \frac{1}{2} \cdot 8 \cdot t^2$

$t = 2$ sec



2. A

Sol. Block is displaced by ‘x’ along x–axis.

Torque about P

$8k(2x_1)\ell = 4k(x - x_1)\frac{\ell}{2}$

$x_1 = \frac{x}{9}$ …(i)

Torque about Q

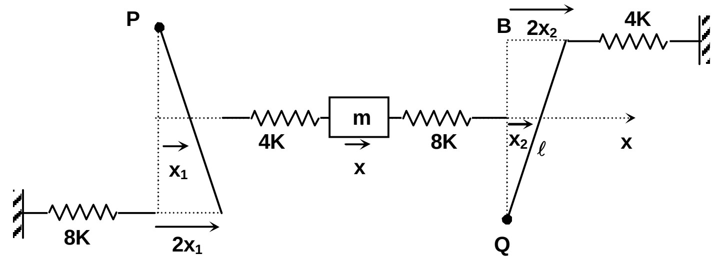
$4k.2x_2.\ell = 8k(x - x_2)\frac{\ell}{2}$

$x_2 = \frac{x}{3}$

$ma = -\frac{80}{9}kx$

$a = -\left(\frac{80k}{9m}\right)x$

$T = 2\pi\sqrt{\frac{9m}{80k}} = \frac{3\pi}{2}\sqrt{\frac{m}{5k}}$



3. C

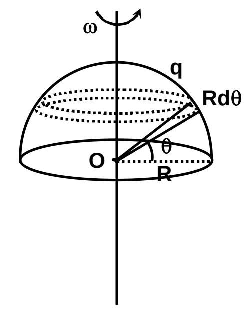
Sol. Magnetic field due to the ring on its axis is

$B = \frac{\mu_0 \pi r^2 i}{2\pi(r^2 + x^2)^{3/2}}$ (due to ring)

$$B = \int_0^{+\pi/2} \frac{\mu_0 \pi (R\cos\theta)^2 \left(\frac{2\pi R\cos\theta R d\theta \sigma}{2\pi/\omega}\right)}{2\pi\left[(R\cos\theta)^2 + (R\sin\theta)^2\right]^{3/2}}$$

Where $\sigma = \frac{q}{2\pi R^2}$

$B = \frac{\mu_0 q \omega}{6\pi R}$



4. A

Sol. $I_{CBF} = (K)\left(A_1^2 + A_2^2 + 2A_1A_2\cos\theta\right) = I_0$

$A_1 = A,\ A_2 = 5A,\ \theta = 0°$

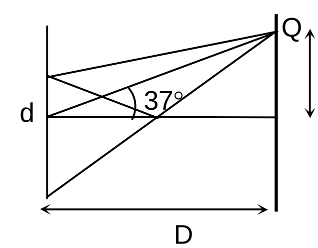
$kA^2 = \frac{I_0}{36}$

$I_Q = I = \frac{16}{25}K\left(A_1^2 + A_2^2 + 2A_1A_2\cos\phi\right)$

$\phi = \frac{2\pi}{\lambda}\Delta x$

$\phi = \frac{2\pi}{\lambda}\cdot\frac{3}{5}d = \frac{\pi}{3}$

$I_Q = \left(\frac{124}{225}\right)I_0$



$\Delta x = d\sin 37°$

$\Delta x = (3/5)d$

5. D

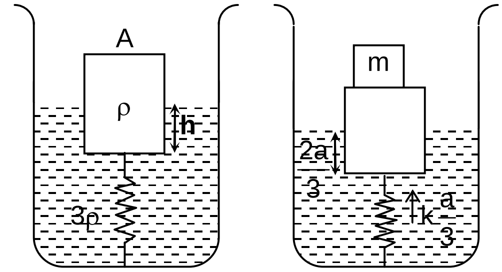
Sol. $(Ah)3\rho g = (Aa)\rho g$

$h = a/3$

$mg + (a^3\rho g) = k\left(\frac{a}{3}\right) + \left(\frac{2}{3}a^3\right)(3\rho)g$

$mg = \frac{4}{3}a^3\rho g$

$m = 4$ kg

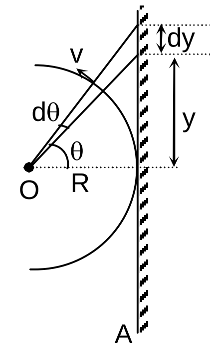


6. D

Sol. $y = R \tan\theta$

$$v_{shadow} = R\sec^2\theta\left(\frac{d\theta}{dt}\right) = v\sec^2\left(\frac{vt}{R}\right)$$

$$a_{shadow} = \frac{dv_{shadow}}{dt} = \frac{2v^2}{R}\sec^2\left(\frac{vt}{R}\right)\tan\left(\frac{vt}{R}\right)$$



7. A, B, C, D

Sol. By conservation of energy

$$\frac{1}{2}mv^2 = mg\frac{\ell}{2} \Rightarrow v = \sqrt{g\ell}$$

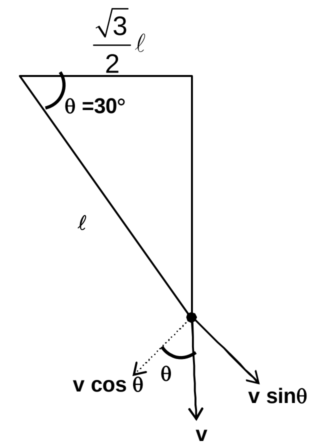
Just after jerk v sin θ becomes zero

Impulse applied by string = mv sin θ = $(0.1)\sqrt{g\ell}\,\frac{1}{2} = \frac{\sqrt{g\ell}}{20}$

Velocity of ball after experiencing jerk $= v\cos\theta = \frac{\sqrt{3g\ell}}{2}$

Velocity at B $v_B^2 = \frac{3g\ell}{4} + 2\cdot g\frac{\ell}{2} = \frac{7}{4}g\ell$

$$v_B = \left(\frac{\sqrt{7g\ell}}{2}\right)$$



8. C, D

Sol. Electric field due to rod

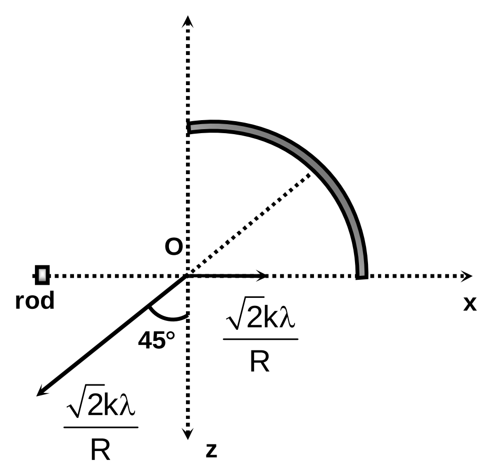
$$E = \frac{k\lambda}{R}(\sin\theta_1 + \sin\theta_2)$$

Electric field due to circular arc

$$E = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)$$

$$E_0 = \left[\sqrt{(\sqrt{2}-1)^2 + (1)^2}\right]\frac{k\lambda}{R}$$

$$E_0 = \left(\frac{\sqrt{4-2\sqrt{2}}}{4\pi\varepsilon_0}\right)\frac{\lambda}{R}$$



9. A, B, C, D

Sol. Focal length of lens

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{2}{R}\right) = \left(\frac{3}{2}-1\right)\left(\frac{2}{20}\right)$$

Position of Image from lens

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$\frac{1}{20} = \frac{1}{v} + \frac{1}{40} \Rightarrow v = 40$ cm

So, position of objet from the concave mirror is 30 which is centre of concave mirror. Hence final image will form at the position of object.

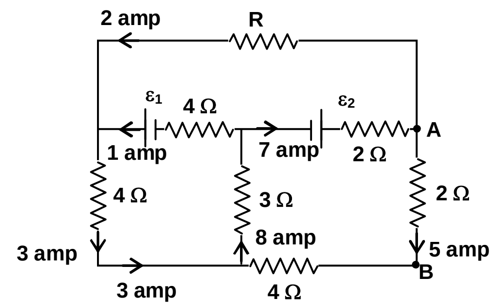
10. A, B, C, D

Sol. By applying Kirchhoff's law

$\varepsilon_1 = 40$ V

$\varepsilon_2 = 68$ V

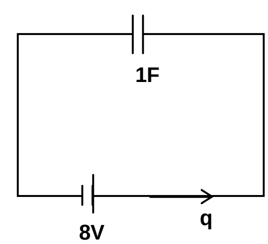
R = 9 Ω and current through 4Ω = 5 amp.



11. A, B, C, D

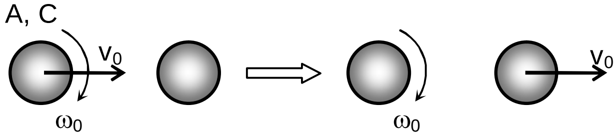
Sol. Equivalent circuit

q is the amount of charge flown in the circuit. q = 8 coulomb.



12. A, C

Sol.



SECTION – C

13. 00001.20

Sol. Time between two consecutive collisions = $\frac{1}{1000}$ sec

So, $\frac{2\ell}{v_{rms}} = \frac{1}{1000}$

$v_{rms} = 2000 \times 5 = 10000$ m/s

$v_{rms} = \sqrt{\frac{3RT}{M}} = 10000$

So, T = $\frac{10^8 \times 4 \times 10^{-3}}{3 \times \left(\frac{25}{3}\right)} = 16 \times 10^3 K$

Mass of helium gas ⇒ PV = nRT

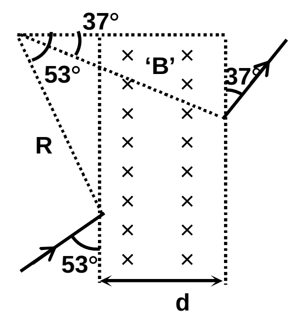
PV = $\frac{m}{M}$RT

$m = \frac{PVM}{RT} = \frac{320 \times 125 \times (4 \times 10^{-3})}{\left(\frac{25}{3}\right) \times 16 \times 10^3} = 1.2\,gm$

14. 00080.00

Sol. $d = R\cos 37^\circ - R\cos 53^\circ$

$d = 80$ cm



15. 00060.00

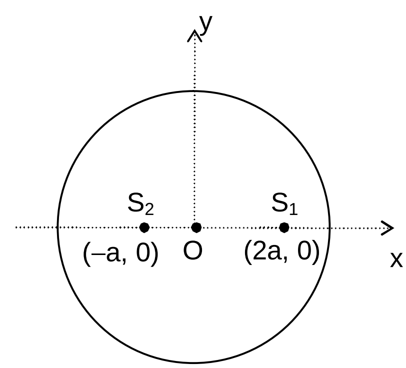
Sol. $\Delta r_{max} = 3a$

$n\lambda = 3a$

$n = \frac{3a}{\lambda}$

$n = \frac{3a}{(a/5)} = 15$

So, total number of maxima detected during one circular revolution = $15 \times 4 = 60$



16. 00100.00

Sol. i_1 is current in the first loop

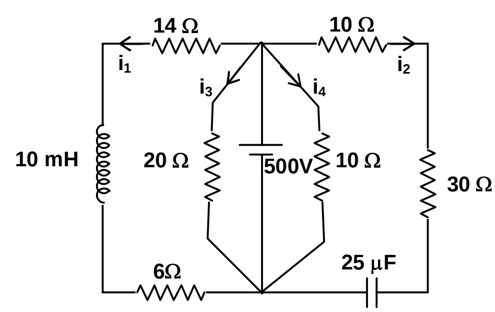
$$i_1 = \frac{500}{20}\left(1 - e^{-\frac{t}{5\times10^{-4}}}\right) = \frac{75}{4} \text{ amp}$$

i_2 is current in the second loop

$$i_2 = \frac{500}{40}e^{-\frac{t}{10^{-3}}} = \frac{25}{4} \text{ amp}$$

$$i_3 = \frac{500}{20} \text{ amp}$$

$$i_4 = \frac{500}{10} \text{ amp}$$



So, total current through key (K)

$i = i_1 + i_2 + i_3 + i_4 = 100$ amp

17. 00004.00

Sol. $T = \frac{F}{\ell}x$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{F}{\ell}\frac{x}{m/\ell}} = \sqrt{\frac{Fx}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{Fx}{m}}$$

$$\int_0^{\ell}\frac{dx}{\sqrt{x}} = \sqrt{\frac{F}{m}}\int_0^t dt$$

$$t = 2\sqrt{\frac{m\ell}{F}}$$

18. 00029.00

Sol. Using Moseley's law

$\sqrt{\nu} = a(Z - 1)$, we have

$$\sqrt{\frac{c}{\lambda_A}} = a(Z_A - 1)$$

and $\sqrt{\frac{c}{\lambda_x}} = a\left(Z_x - 1\right)$

Dividing yields $\sqrt{\frac{\lambda_A}{\lambda_x}} = \frac{Z_x - 1}{Z_A - 1}$

$\Rightarrow Z_x = 29.$

Chemistry

PART – II

SECTION – A

19. A
Sol. The number of maxima in the radial charge density curve is (n – *l*)
For 3d electron, n = 3 and d = 2
Hence, n – *l* = 3 – 2 = 1
There will be only one maximum in the curve for 3 d electron.

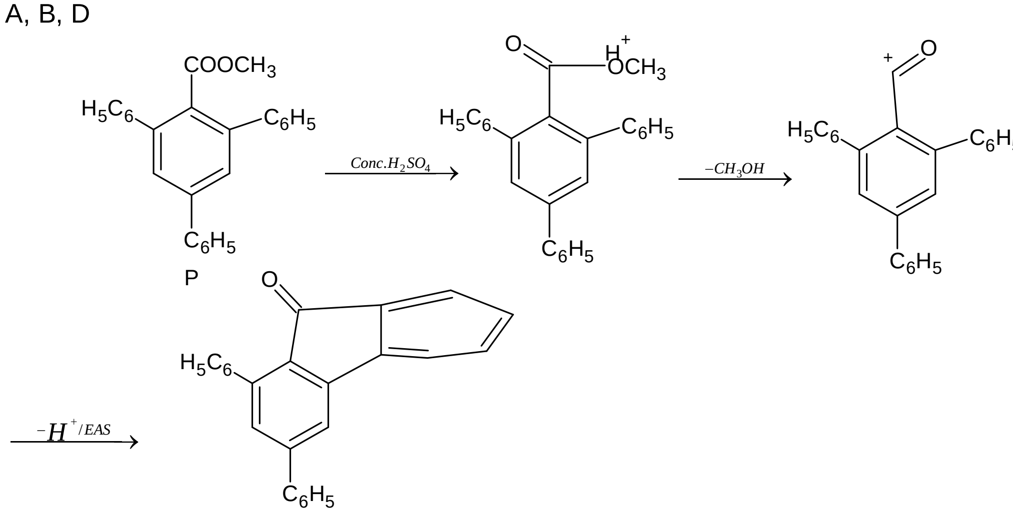
20. C
Sol. The order of ligand strength in the spectrochemical series $F^- < H_2O < NH_3 < NO_2^-$.
And CFSE for octahedral complex is $Fe^{+2} < Co^{+2}$ due to high ENC . A strong ligand causes a larger degree of splitting resulting in high value of E (energy). Therefore, corresponding low value of $\lambda \left[E = \frac{hc}{\lambda}\right]$

21. A
Sol. **Rate =**$k[A]^{\alpha}[B]^{\beta}$ Using the given data α=1,β = 2 overall order is 3 and reaction may or may not be elementary.

22. B
Sol. Fact based

23. B
Sol. $$\underset{}{Mg_3N_2} + H_2O \longrightarrow Mg(OH)_2 + \underset{(A)}{NH_3}$$
$$NH_3 + CuO \longrightarrow Cu + \underset{(C)}{N_2} + H_2O$$
Ammonium salts containing anions which are oxidizing in nature produces nitrogen on heating ammonium

24. A
Sol. The given equation is for one mol van der waal's gas.
If P ext =P means process is reversible and this expression is not valid for any irreversible process.

25. A, B, D
Sol.


X

Compound X is aromatic in nature and it has 5 cyclic ring, 4 are 6-membered and 1 is 5-membered ring and it can react with 1mol Grignard reagent.

26. B,C,D

Sol. Its BCC unit cell having rank 2 can be compared with CsCl structure so radius of M to X is 0.732:1.00 .CN for both is 8 nearest distance between M and X is a$\sqrt{3}$/2=0.866a.

27. B, C, D

Sol. Zeise's salt $K\left[PtCl_3C_2H_4\right]$ has Square planar geometry. $\left[Co(en)(NH_3)2Cl_2\right]^+$ is inner orbital octahedral d^2sp^3 complex and has 3 G.I and 2 optical isomers. $\left[FeCl_4\right]^-$ has higher spin-only magnetic moment as it has 5 unpaired electron than $\left[Co(en)(NH_3)2Cl_2\right]^+$ which has only 1 unpaired electron. In option (D) The cobalt ion in $\left[Co(en)_2(NH3)_2\right]^{2+}$ has d^2sp^3 hybridization as both act as SFL for Co^{+2}.

28. B,C,D

Sol. Draw the bond line structure of each molecule you will get to know (P, Q) and (R, S) Represents different set of molecules while Q and R are same molecules. All the given molecules has no chiral centre so all are achiral. None of the molecule will be oxidized by acidic potassium dichromate solutions as all are tertiary alcohols.

29. C, D

Sol. All reducing sugars are mutarotating. Although IV and VI are an α-hydroxy ketone and hence reducing but it can't mutarorate as it is not a carbohydrate/can't form ring. In II the glycosidic bond is in between two anomeric carbons and hence ring opening can't occur. Thus non-reducing as well as non-mutarotating.

30. A. B, D

Sol. The white pt. (B) finally formed is $PbSO_4$. The lead iodate on heated t Liberates both I_2 and O_2 n-factor of Iodate while heating is 10 and The brown gas liberated is NO2 which is paramagnetic in nature.

$$\underset{\text{Black}}{PbS}\downarrow \xrightarrow{H_2O_2} \underset{\text{White}}{PbSO_4}\downarrow$$

$$Pb(IO_3)_2 \xrightarrow{\Delta} PbO + I_2 + O_2$$

SECTION – C

31. 00001.40

Sol. Let the no. of moles of H_2, O_2 and He are x, y and z respectively

$\therefore$ x + y + z = 2.0

Pressure exerted by 2 moles of gaseous mixture is 50 atm.

After the first electric spark decrease in pressure = 50 – 12.5 = 37.5 atm

$\therefore$ decrease in no. of moles of gaseous mixture is 1.5

But from the given information limiting reagent in first step is O_2

$\therefore$ from $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell)$

$\therefore$ H_2 and O_2 reacted are in the ratio of 2 : 1

$\therefore$ 0.5 mole of O_2 should be reacted with 1.0 mole of H_2

$\therefore$ No. of mole of O_2 in the mixture (y) = 0.5

When again O_2 is passed, pressure increased from 12.5 to 25.0 atm

$\therefore$ Change in pressure = 12.5 atm

$\therefore$ No. of moles of O_2 added = $\frac{12.5\times 2}{50} = 0.5$ mole

Now H_2 will be completely reacted

No. of moles H_2 actually left after first electric spark = x – 1

$H_2 \quad + \quad \frac{1}{2}O_2 \quad \longrightarrow H_2O$

$(x-1) \quad \left(\frac{x-1}{2}\right)$

Now change in pressure in 25 – 10 = 15 atm

$\therefore$ change in no. of moles is $\frac{15\times 2}{50} = 0.6$

$\therefore \ (x-1)+\left(\frac{x-1}{2}\right) = 0.6$

$\Rightarrow x = 1.4$

$\therefore$ no. of moles of H_2 = 1.4

32. 00027.20

Sol. We need to calculate on the same day, one leap year later, so, number of days, nanoturtle got to grow is 366.

So, $d_{366} = d_0 \times \ e^{+2.732\times 10^{-3}\times t}$

$= 10\,\overset{o}{A} \times e^{+2.732\times 10^{-3}\times 366}$

$= \left(10\,\overset{o}{A}\right)\times e^{1} = 10\times 2.720 = 27.20$

33. 00001.75

Sol. For isoentropic process $\Delta_{system} = 0$

$\therefore nC_P, m\ell n\left(\frac{T_2}{T_1}\right) + nR\ell n\frac{P_1}{P_2} = 0$

$\Rightarrow \ell n(P_2) = \left(\frac{5}{2}\right)\times \ell n\left(\frac{600}{300}\right) = 1.75$ atm

34. 00001.73

Sol. (i) $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O \qquad (\Delta G_1^0)$

(ii) $MnO_2 + 4H^+ + 2e^- \longrightarrow Mn^{2+} + 2H_2O \qquad (\Delta G_2^0)$

Equation (i) and (ii) give the required equation

$MnO_4^- + 4H^+ + 3e^- \longrightarrow MnO_2 + 2H_2O \qquad (\Delta G_3^0)$

$\Delta G_3^0 = \Delta G_1^0 - \Delta G_2^0$

$-3FE_3^0 = -5FE_1^0 - (-2FE_2^0)$

$E_3^0 = \frac{5E_1^0 - 2E_2^0}{3}$

$E_3^0 = \frac{(5\times 1.51)-(2\times 1.18)}{3} = 1.73$ V.

35. 00007.53

Sol. (a) on mixing into (i)

$PO_4^{3-} + H_2PO_4^- \rightleftharpoons 2HPO_4^{2-}$

	PO_4^{3-}	$H_2PO_4^-$	$2HPO_4^{2-}$
	10	10	0
at eqbm	0	0	20

(b) on mixing this into (ii)

$$HPO_4^{2-} + OH^- \rightleftharpoons PO_4^{3-}$$

	20	10	0
at eqbm	10	0	10

(c) Now, on mixing this into (iii)

$$PO_4^{3-} + H_2PO_4^- \rightleftharpoons 2HPO_4^{2-}$$

	10	10	0
at eqbm	0	0	10

$$HPO_4^{2-} + H^+ \rightleftharpoons H_2PO_4^-$$

	20	15	0
at eqbm	5	0	15

So, final mixture is a buffer.

Hence, $pH = pK_{a_2} - \log_{10} 3 = 7.53$

36. 00018.75

Sol.

$$A \longrightarrow 2B + C$$

$2 - x \qquad 2x \qquad x$

$$P_s = \frac{20}{22 + 2x} \times 24 \qquad (\text{at } t = 12 \text{ hr})$$

$x = 1$ mole which is half of the initial value.

Hence, 12 hour is half-life for the above first order reaction.

The above reaction will be completed 99% in 80 hrs.

$$P_s = \frac{20}{22 + 2 \times 1.8} \times 24 \qquad (\text{at } t = 80 \text{ hr})$$

$P_s = 18.75$ mm Hg

Mathematics

PART – III

SECTION – A

37. C

Sol. Given $2\frac{dy}{dx} = \frac{3}{x^2} - y^2 = \frac{3-(xy)^2}{x^2}$

Let t = xy $\Rightarrow x\frac{dy}{dx} + y = \frac{dt}{dx}$

$\Rightarrow x\left(\frac{1}{2}\left(\frac{3-t^2}{x^2}\right)\right) + \frac{t}{x} = \frac{dt}{dx} \Rightarrow \frac{3-t^2}{2x} + \frac{t}{x} = \frac{dt}{dx}$

$\Rightarrow \int \frac{-dt}{t^2-2t-3} = \int \frac{dx}{2x} \Rightarrow \frac{1}{2}\left(\ln\left|\frac{t+1}{t-3}\right|\right) = \ln|x| + \ln C$

$\Rightarrow \sqrt{\left|\frac{t+1}{t-3}\right|} = C|x| \Rightarrow \frac{xy+1}{xy-3} = kx^2$ passes through (–1)

$\Rightarrow$ k = 0 $\Rightarrow$ curve C is y = $-\frac{1}{x}$

38. B

Sol. Let $A(x_1, y_1)$, $B(x_1, y_2)$, $C(t^2, 2t)$ from $x - 2y - 1 = 0$ and $y^2 = 4x$, we get $y^2 - 8y - 4 = 0$

$y_1 + y_2 = 8$ and $y_1y_2 = -4$

$x_1 = 2y_1 + 1$, $x_2 = 2y_2 + 1$

$x_1 + x_2 = 2(y_1 + y_2) + 2 = 18$

$x_1 \cdot x_2 = 2(y_1 + y_2) + 4y_1y_2 + 1 = 1$

$\angle ACB = 90^\circ \Rightarrow m_{AC} \cdot m_{BC} = -1 \Rightarrow (t^2 - x_1)(t^2 - x_2) + (2t - y_1)(2t - y_2) = 0$

$\Rightarrow t^4 - 14t^2 - 16t - 3 = 0 \Rightarrow t^2 + 4t + 3 = 0$ or $t^2 - 4t - 1 = 0$

t = –1, –3

Coordinate of C is (1, –2) or (9, –6)

$t^2 - 4t - 1 = 0$ not possible as C. Coincides with A or B

39. D

Sol. For $2^i \le k \le 2^{i+1} - 1$; $[\log_2 k] = i$

So, $\sum_{k=1}^{2^n}[\log_2 k] = [\log_2 1] + [\log_2 2].....[\log_2 2^n]$

$= 0 + 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 \ldots\ldots (n-1)\cdot 2^{n-1} + n$

$= (n-2)\cdot 2^n + n + 2$

40. D

Sol. $|PF_1| + |PF_2| = 2a = 12$ by definition of an ellipse, since $\frac{|PF_1|}{|PF_2|} = \frac{2}{1}$

Then $|PF_1| = 8$; and $|PF_2| = 4$ and $|F_1F_2| = 2ae = 12\sqrt{1-\frac{16}{36}} = 4\sqrt{5}$

and $|PF_1|^2 + |PF_2|^2 = 8^2 + 4^2 = 80$

Then ΔPF_1F_2 is a right $= |F_1F_2|^2$ triangle

So, $PF_1F_2 = \frac{1}{2}|PF_1||PF_2| = \frac{1}{2} \times 8 \times 4 = 16$

41. C

Sol. Equation of plane P is $\left(\vec{r} - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\right)\cdot\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right) = 0$

42. A

Sol. Obviously $f(x) = x^3 + \log_2\left(x + \sqrt{x^2+1}\right)$ is odd function and is monotonically increasing

So if a + b ≥ 0, i.e. a ≥ –b, we get f(a) ≥ f(–b) > f(a) ≥ –f(b)

⇒ f(a) + f(b) ≥ 0

On other hand if f(a) + f(b) ≥ 0, then f(a) ≥ –f(b)

So, a ≥ –b, a + b ≥ 0

43. A, B, C

Sol. Every element of A appears 3 times. Then we have

$3(a_1 + a_2 + a_3 + a_4) = -1 + 3 + 5 + 8 = 5 \Rightarrow a_1 + a_2 + a_3 + a_4 = 5$

A = {–3, 0, 2, 6}

44. A, D

Sol. $\Delta = b^2c^2 - 4a(b^3 + c^3 - 4abc) = (b^2 - 4ac)(c^2 - 4ab) < 0$

So, any one of b^2 – 4ac and c^2 – 4ab is less than zero and other is greater than zero

45. A, B

Sol. $e^x\,dx + xe^x\,dx - xe^x\,dy + ye^y\,dy = 0$

$\Rightarrow e^x\,dx + xe^x(dx - dy) + ye^y\,dy = 0$

$\Rightarrow e^{x-y}\,dx + xe^{x-y}(dx - dy) + y\,dy = 0$

$\Rightarrow d(xe^{x-y}) + d\left(\frac{y^2}{2}\right) = 0$

$\Rightarrow xe^{x-y} + \frac{y^2}{2} = c$ as f(1) = 1 $\Rightarrow c = \frac{3}{2}$

$xe^{x-y} + \frac{y^2}{2} = \frac{3}{2}$. So, value of f(0) is $\pm\sqrt{3}$

46. A, B, C

Sol. Let r < n, then total number of ways of distributing r differents things to n has as such that each should get atleast one things is

$n^r - {}^nC_1(n-1)^r + {}^nC_2(n-2)^r + ...(-1)^{n-1}\cdot{}^nC_{n-1} = 0$

Let r = n

$n^n - {}^nC_1(n-1)^n + {}^nC_2(n-2)^n - .....(-1)^{n-1}\cdot{}^nC_{n-1} = n!$

$\Rightarrow {}^nC_1 - {}^nC_2 2^n + {}^nC_3 3^n - .... = (-1)^{n-1}\cdot n!$

47. C, D

Sol. Let common difference of $\{a_n\}$ is d, common ratio of $\{b_n\}$ is r, then

3 + d = r ….. (1)

$3(3 + 4d) = r^2$ ….. (2)

⇒ r = 9, d = 6

$\Rightarrow a_n = \log_\alpha^{b_n} + \beta$

$\Rightarrow 3 + (n-1)6 = \log_\alpha^{9^{n-1}} + \beta$

$\Rightarrow 3 + (n-1)6 = 2(n-1)\log_\alpha^3 + \beta$ hold for every positive integer n

$\Rightarrow \alpha = (3)^{\frac{1}{3}}, \beta = 3$

48. B, C

Sol. From the given inequality $\cos^5\theta - \sin^5\theta < 7(\sin^3\theta - \cos^3\theta)$

$\sin^3\theta + \frac{1}{7}\sin^5\theta > \cos^3\theta + \frac{1}{7}\cos^5\theta$

$\therefore\ f(x) = x^3 + \frac{1}{7}x^5$ is increasing over R, then sin θ > cos θ

and that mean $2k\pi + \frac{\pi}{4} < \theta < 2k\pi + \frac{5\pi}{4}$ $(k \in I)$

So, range of $\theta \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

SECTION – C

49. 00001.00

Sol. af(x) + bf(x – c) = a(3 sin x + 2 cos x + 1) + b(3 sin (x – c) + 2 cos(x – c) + 1) = 1

So, $c = (2n+1)\bar{x}$ $x \in Z$ and a = b = $\frac{1}{2}$; $\frac{b\cos c}{a} = -1$

50. 00011.00

Sol. f′(x) = 3ax^2 + 2bx + c ; f′(0) = c, $f'\left(\frac{1}{2}\right) = \frac{3}{4}a + b + c$; f′(1) = 3a + 2b + c, then

$3a = 2f'(0) + 2f'(1) - 4f'\left(\frac{1}{2}\right)$, we get $3|a| = \left|2f'(0) + 2f'(1) - 4f'\left(\frac{1}{2}\right)\right| \le 2|f'(0)| + 2|f'(1)| + 4\left|f'\left(\frac{1}{2}\right)\right|$

$3|a| \le 8$, $|a| \le \frac{8}{3}$, $a \in \left[-\frac{8}{3}, \frac{8}{3}\right]$. So, p + q = 11

51. 00011.00

Sol. sin A – cos A = 10[sin B sin C – cos B cos C] = –10 cos (B + C) = 10 cos A

$\Rightarrow$ tan A = 11

52. 00000.50

Sol. $\frac{a_1^2 + a_2^2 + a_3^2}{b_1 + b_2 + b_3} = \frac{14}{1 + r + r^2}$

Possible if $r = \frac{1}{2}$

53. 00232.00

Sol. Suppose $a_1 < a_2 < a_3 < a_4 < a_5$ taken from 1, 2, ….., 20.

If a_1, a_2, a_3, a_4, a_5 are not adjacent to each other then, we have

$1 \le a_1 < a_2 - 1 < a_3 - 2 < a_4 - 3 < a_5 - 4 \le 16$

From which we known that number of ways to select five numbers not adjacent to each other from 1 to 20 is the same as selecting five different numbers from 1 to 16

So, required probability = $\frac{^{16}C_5}{^{20}C_5} = \frac{232}{323}$

54. 00002.00

Sol. Let altitude, angle bisector and median from vertex C meet the side AB at D, E and F respectively

Now, $\frac{AE}{EB} = \frac{AC}{CB} = \frac{b}{a} \Rightarrow AE = \frac{bc}{a+b}$

Also, $\frac{FB}{FE} = \frac{BC}{CE} = \frac{a}{b} \Rightarrow FB = \frac{c}{2}$ and $FE = \frac{c(a-b)}{2(a+b)}$

$\Rightarrow \left(\frac{b}{a}\right)^2 + 2\left(\frac{b}{a}\right) - a = 0 \Rightarrow \frac{b}{a} = \sqrt{2} - 1 \Rightarrow \angle C = 90°$