FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Main)-2019

TEST DATE: 9-12-2018

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. B

Sol. Basic Concepts

2. C

Sol. $W = A\int_0^{V_0/2A}(P_0 + \rho gy)dy = \frac{P_0V_0}{2} + \frac{\rho gA}{2}\left(\frac{V_0}{2A}\right)^2 = \frac{5P_0V_0}{8}$

Also $n = \frac{P_0V_0}{RT_0} = \frac{\left(P_0 + \rho g\frac{V_0}{2A}\right)\frac{3V_0}{2}}{RT} \Rightarrow T = \frac{\frac{3}{2}\left(P_0 + \rho g\frac{V_0}{2A}\right)T_0}{P_0} = \frac{9T_0}{4}$

$\therefore \quad \Delta U = nC_v\Delta T = \left(\frac{P_0V_0}{RT_0}\right)\left(\frac{3R}{2}\right)\left(\frac{9T_0}{4} - T_0\right) = \frac{15}{8}P_0V_0$

$\therefore \quad \frac{\Delta U}{\Delta W} = 3.$

3. A

Sol. $\frac{\lambda_A}{\lambda_B} = \left(\frac{Z_B - 1}{Z_A - 1}\right)^2$

4. C

Sol. $V_0 = \sqrt{\frac{GM}{R}}$

Binding energy of satellite = $\frac{GMm}{2R}$

If V_e is escape speed, then $\frac{1}{2}mV_e^2 = \frac{1}{2}mV_0^2 + \frac{GMm}{2R}$

$\Rightarrow \quad V_e = \sqrt{2}\left[\frac{GM}{R}\right]^{1/2} = \sqrt{2}V_0$

Using conversation of momentum;

$mV_0 = (m - \Delta m)V_e - \Delta m U_{max}$

$\Rightarrow \quad mV_0 = \left[m - \frac{5}{100}m\right]\sqrt{2}V_0 - \frac{5m}{100}U_{max}$

$U_{max} = \frac{(\sqrt{2}\times 95 - 100)}{5}V_0 \approx 7V_0$

5. C

Sol. $\frac{v_1}{v_2} = \frac{27}{8}$

So, $\frac{m_1}{m_2} = \frac{8}{27}$ (from conservation of momentum)

$r \propto A^{1/3}$

$\frac{r_1}{r_2} = \left(\frac{8}{27}\right)^{1/3} = \frac{2}{3}$

6. B

Sol. $7\times\frac{D\lambda}{d} = \frac{(\mu_1 - \mu_2)t\cdot D}{d}$

$t = 9.1\ \mu m$

7. D

Sol. $E \propto \frac{1}{n^2}$

$r \propto n^2$

$p \propto \frac{1}{n}$

$L \propto n$

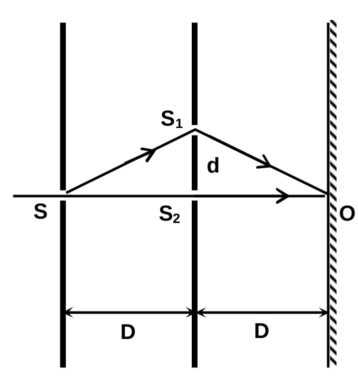
8. B

Sol. path difference = $(SS_1 + S_1O) - SO$

$= 2\sqrt{D^2 + d^2} - 2D = \frac{d^2}{D}$

$\frac{d^2}{D} = n\lambda$

$d = \sqrt{D\lambda}$ (n = 1)



9. A

Sol. Let plank is slight displacement a distance x.

$$F_r = \frac{32}{3}kx$$

$$a = \frac{32k}{3m}x$$

$$\omega = \sqrt{\frac{32k}{3m}}$$

10. C

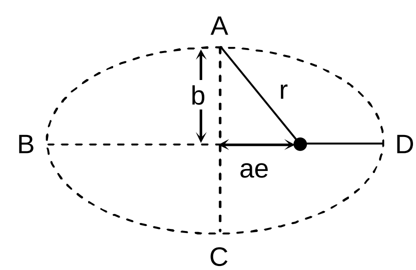
Sol. $v = LT^{-1}$
$G = M^{-1}L^3T^{-2}$
$h = ML^2T^{-1}$
So, $\frac{v}{h} = \frac{1}{ML}$
and $\frac{G}{h^2} = \frac{1}{M^3L}$
Hence, $L = G^{1/2}, h^{1/2}, v^{-3/2}$

11. D

Sol. $r^2 = b^2 + a^2e^2$ …(1)
$b^2 = a^2\left(1-e^2\right)$ …(2)
From equation (1) & (2)
r = a
Now from conservation of energy

$$-\frac{GMm}{2a} = \frac{1}{2}mv^2 - \frac{GMm}{a}$$

$$v = \sqrt{\frac{GM}{a}}$$

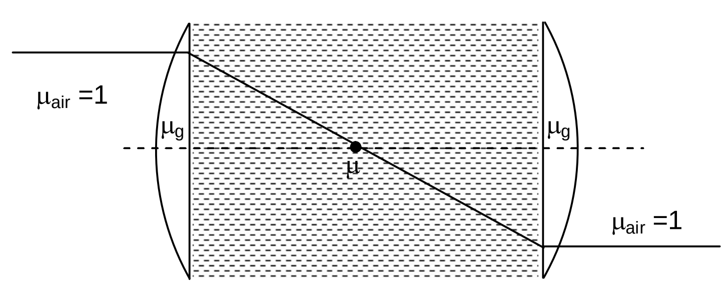


12. B

Sol. $$\frac{\mu}{d/2} - \frac{1}{\infty} = \frac{\mu_g - 1}{R} - \frac{\mu_g - \mu}{\infty}$$

$$\Rightarrow \quad d = \frac{2\mu R}{\mu_0 - 1} = 2\mu f$$

$$\left(\text{where} \frac{1}{f} = \frac{\mu_0 - 1}{R}\right)$$



13. C

Sol. $$(3\rho)VgR\theta - \frac{V}{2}2\rho g\frac{5R}{8}\theta - \frac{V}{2}\rho g\frac{11R}{8}\theta = \left[2\rho\frac{V}{2}R^2\left(\frac{2}{5} - \frac{9}{64} + \frac{25}{64}\right) + \rho\frac{V}{2}R^2\left(\frac{2}{5} - \frac{9}{64} + \frac{121}{64}\right)\right]\alpha$$

So $\omega = \sqrt{\frac{45g}{46R}}$.

14. A

Sol. $f' = f_0\left(\frac{1 - v_1/v_s}{1 - v_2/v_s}\right)$, $v_1 = 0$ and $v_2 = -\omega R$

15. A

Sol. Gravitation potential due to disc is

$V = -\frac{2Gm}{R^2}(\sqrt{R^2 + \ell^2} - \ell)$

So, $U_i = -\frac{Gmm}{\ell}$ $(R << \ell)$

$U_f = -\frac{2Gmm}{R}$ $(\ell \to 0)$

$\Delta U = U_i - U_f = Gmm\left(\frac{2}{R} - \frac{1}{\ell}\right)$

$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = Gmm\left(\frac{2}{R} - \frac{1}{\ell}\right)$

$v = \sqrt{Gm\left(\frac{2}{R} - \frac{1}{\ell}\right)}$

So, relative velocity

$v_R = 2v = \sqrt{4Gm\left(\frac{2}{R} - \frac{1}{\ell}\right)}$

16. B

Sol. $T = 2\pi\sqrt{\frac{I}{mgd}}$

$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$

$f_0 = \frac{1}{2\pi}\sqrt{\frac{(2m)(g)(3\ell/4)}{\left(\frac{m\ell^2}{3} + m\ell^2\right)}} = \frac{1}{2\pi}\sqrt{\frac{9g}{8\ell}}$

When disc is removed

$f = \frac{1}{2\pi}\sqrt{\frac{mg(\ell/2)}{m\ell^2/3}} = \frac{1}{2\pi}\sqrt{\frac{3g}{2\ell}}$

$\frac{f}{f_0} = \sqrt{\frac{3}{2} \times \frac{8}{9}} = \frac{2}{\sqrt{3}}$

17. D

Sol. The maximum allowable zener current = $\frac{0.36}{12} = 0.03A = 30$ mA

Case I: If $R_L \to \infty \Rightarrow V_R = V - V_2 = 15 - 12 = 3$ volt $\Rightarrow R = \frac{V_R}{I_2} = \frac{3}{0.03} = 100\Omega$

Case II: If R_L is finite

$I = I_L + I_2$

As $R_L\downarrow \Rightarrow I_L\uparrow \Rightarrow I_2\downarrow$, so for minimum value of R_L, the I_2 will be 2mA, so $I_L = 30 - 2 = 28$ mA

$R_{L_{min}} = \frac{12}{I_L} = \frac{12}{0.028} = 430\Omega \Rightarrow 430\ \Omega \le R_L < \infty$

18. C

Sol. The amplitude is doubled, the intensity is quadrupled.

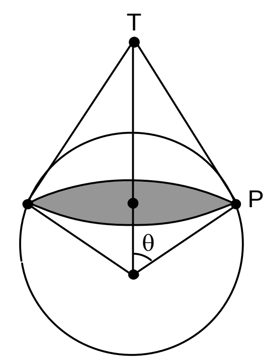
19. C

Sol. To cover $\left(\frac{1}{4}\right)^{th}$ of the earth's surface, the direct transmission reaches a point 'θ' from the transmitters where:

$2\pi(1-\cos\theta)R^2 = \frac{1}{4}.4\pi R^2$

$\therefore \theta = 60°$.

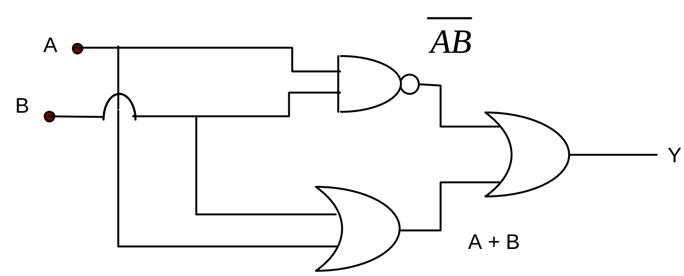
$\therefore$ height, h = 2R – R = R



20. D

Sol. $Y = A + B + \overline{AB}$

$\Rightarrow Y = A + B + \overline{A} + \overline{B} = 1$



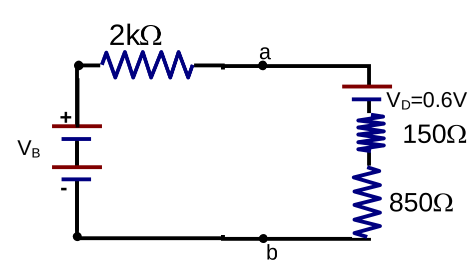
21. A

Sol. As shown in the Figure, the diode is replaced by its equivalent circuit and the circuit to the left of the terminals a, b.

Since the diode can dissipate a maximum power of 200 mW, the maximum safe diode current I will satisfy the relationship

$P = 200 \times 10^{-3} = i^2 r = 150 i^2$

$\Rightarrow i = \sqrt{\frac{0.2}{150}} = 0.0365\,A = 36.5$ mA.



As shown in the Figure, $i = \frac{(V_B/3) - 0.6}{3} = 36.5 \Rightarrow V_B = 330$ V,

Which is the maximum permissible battery voltage.

22. B

Sol. $Y = \frac{4F\ell}{\pi d^2 \Delta\ell}$

$\therefore \frac{\Delta Y}{Y} = -\left(2\frac{\Delta d}{d} + \frac{\Delta(\Delta\ell)}{\Delta\ell}\right) = \pm\left(2\times\frac{0.01}{0.4} + \frac{0.05}{0.8}\right) = \pm\frac{9}{80}$

$\therefore$ Y = (2 ± 0.2)×10^{11} N/m^2

23. A

Sol. E = energy stored in the string = $\mu A^2\omega^2 \int_0^{\Delta\pi} \cos^2 kx dx$

Where Δx is distance traveled by the wave in the $\frac{\pi}{12\omega} = \frac{\pi}{12\omega}\frac{\omega\lambda}{2\pi} = \frac{\lambda}{24}$

So, $E = \frac{(\pi+3)\mu A^2\omega^2}{24k}$

24. A

Sol. Let the speed of the flow be v and the diameter of the tap be d = 1.25 cm. The volume of the water flowing out per second is

$Q = v\times\pi d^2 / 4$

$v = 4Q / \pi d^2$

We then estimate the Reynolds number to be

$R_e = 4\rho Q / \pi d\eta$

$= 4\times10^3 \times Q / \left(3.14\times1.25\times10^{-2}\times10^{-3}\right)$

$= 1.109\times10^8 Q$.

For x = 3, $R_e = 5100$ and for x = 6, $R_e = 10200$

and for others it is much lower than the critical value.

25. C

Sol. Now, from equation the time, $T_{1/2}$, for the amplitude to drop to half of its initial value is given by,

$T_{1/2} = -2\pi\frac{\ln(1/2)}{b/2m}$

$= \frac{0.693}{40}\times2\times200\,s \approx 7$ s

26. B

Sol. The linear distance between two dots is $\ell = \frac{2.54}{100}cm \simeq 2.54\times10^{-2}cm$.

At a distance of Z cm this subtends an angle $\phi \sim \ell / z \;\therefore z = \frac{1}{\phi} = \frac{2.54\times10^{-2}cm}{6\times10^{-4}} \approx 45$ cm.

27. C

Sol. Without P:

$A = A_\perp + A_{||}$

$$A_\perp = A_\perp^1 + A_\perp^2 = A_\perp^0 \sin(kx - \omega t) + A_\perp^0 \sin(kx - \omega t + \phi)$$

$$A_{||} = A_{||}^{(1)} + A_{||}^{(2)}$$

$$A_{||} = A_{||}^0 \left[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)\right]$$

Where $A_\perp^0, A_{||}^0$ are the amplitudes of either of the beam in $\perp$ and || polarizations.

$\therefore$ Intensity =

$$= \left\{\left|A_\perp^0\right|^2 + \left|A_{||}^0\right|^2\right\}\left[\sin^2(kx - \omega t)(1 + \cos^2\phi + 2\sin\phi) + \sin^2(kx - \omega t)\sin^2\phi\right]_{average}$$

$$= \left\{\left|A_\perp^0\right|^2 + \left|A_{||}^0\right|^2\right\}\left(\frac{1}{2}\right).2(1 + \cos\phi)$$

$$= 2\left|A_\perp^0\right|^2.(1 + \cos\phi) \text{ since } \left|A_\perp^0\right|_{average} = \left|A_{||}^0\right|_{average}$$

With P:

Assume $A_\perp^0$ is blocked:

Intensity = $\left(A_{||}^1 + A_{||}^2\right)^2 + \left(A_\perp^1\right)^2$

$$= \left|A_\perp^0\right|^2(1 + \cos\phi) + \left|A_\perp^0\right|^2.\frac{1}{2}$$

Given $I_0 = 4\left|A_\perp^0\right|^2 =$ Intensity without polarizer at principal maxima.

Intensity at principal maxima with polarizer

$$= \left|A_\perp^0\right|^2\left(2 + \frac{1}{2}\right) = \frac{5}{8}I_0$$

Intensity at first minima with polarizer

$$= \left|A_\perp^0\right|^2 a(1 - 1) + \frac{\left|A_\perp^0\right|^2}{2} = \frac{I_0}{8}.$$

28. B

Sol. P = Fv = $6\pi\eta r\left[\frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g\right]^2$

$$= \frac{8\pi g^2}{27\eta}(\rho - \sigma)^2 r^5$$

29. C

Sol. Fraction depends only upon the critical angle for the medium.

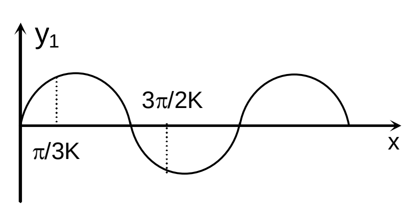
30. D

Sol. At t = 0, shape of standing wave is

So, $\Delta\phi_1 = \pi$

Phase difference $\Delta\phi_2 = k\left(\frac{3\pi}{2k} - \frac{\pi}{3k}\right) = \frac{7\pi}{6}$

$$\frac{\Delta\phi_1}{\Delta\phi_2} = \frac{6}{7}$$



Chemistry PART – II

SECTION – A

31. D

Sol. The mixing of two oppositely charge sols cause coagulation.

32. C

Sol. α-black phosphorous is formed when red phosphorous is heated in a sealed tube at 803 K.

33. D

Sol. $\frac{r_{A^+}}{r_{X^-}} = \frac{1}{2} = 0.5$, it lies in range 0.414 – 0.732, AX has structure like that of NaCl.

Hence, edge length $(a) = 2(r^+ + r^-)$

= 2 (1 + 2) pm

= 6 pm

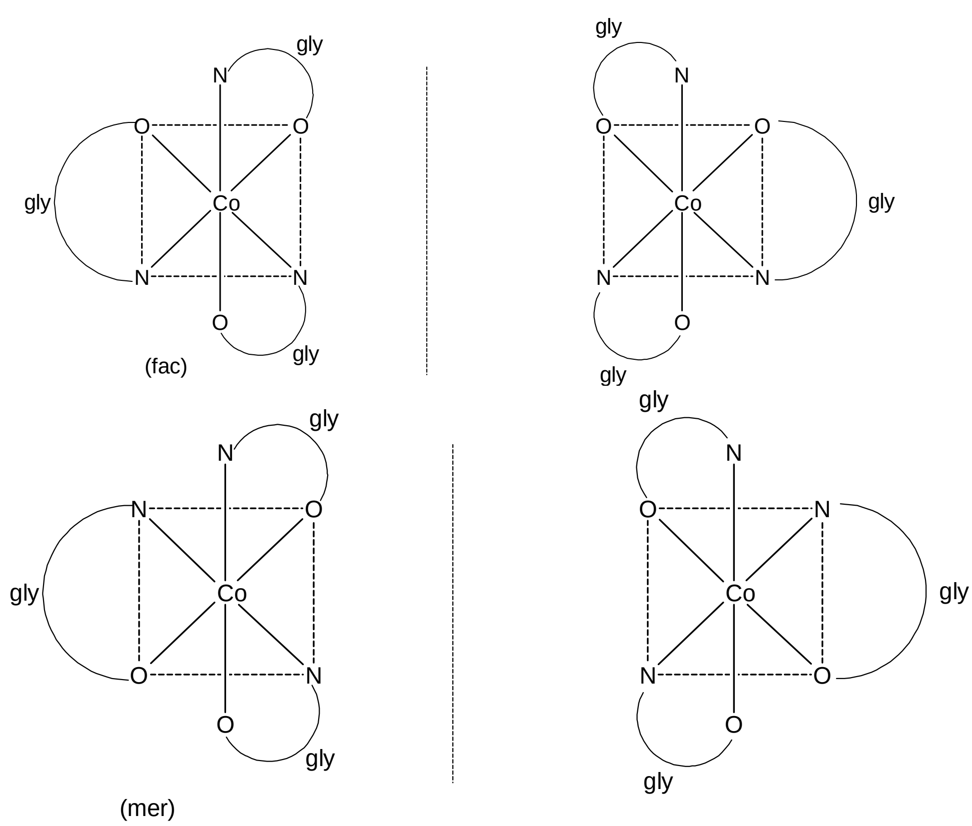
Volume of unit cell = a^3 $(6\ pm)^3$ = 216 pm^3

34. C

Sol. They have high melting point, are hard and chemically inert.

35. B

Sol.



36. B

Sol. Removal of water from $[Ti(H_2O)_6]\,Cl_3$ on heating render it colourless.

$$\underset{\text{Green}}{\left[Ni(H_2O)_6\right]^{2+}} + 3(en)(aq) \longrightarrow \underset{\text{Violet}}{\left[Ni(en)_3\right]^{2+}}(aq) + 6H_2O$$

37. C

Sol. $Ca \rightarrow 1$
$O \rightarrow 3$
$Ti \rightarrow 1$
So, compound is $CaTiO_3$
Oxidation state of Ti is
$+ 2 + x - 6 = 0$
$x = + 4.$

38. C

Sol. $$Ni + 4CO \xrightarrow{330-350\ K} \underbrace{Ni(CO)_4}_{\text{Volatile complex}}$$

39. A

Sol. Millimoles of NaCl = 4 m mole
100 ml requires = 4 m mole
1000 ml requires 40 m mole and this is flocculation value.

40. C

Sol. Equivalents of $K_2Cr_2O_7$ = Equivalents of Fe^{2+}
$M \times V \times nf = n \times nf$
$2 \times V \times 6 = n$
$12 \times V = n$
Equivalents of $KMnO_4$ = Equivalents of Fe^{2+}
$M_1 \times V_1 \times nf_1 = n \times nf$
$2 \times V \times 5 = n \times 1$
$10\ V = n$

41. A

Sol. $P(Vm - b) = RT$

$$\frac{P}{RT} = \frac{1}{Vm - b}$$

$$Z = \frac{PVm}{RT} = \frac{Vm}{Vm - b} = \frac{1}{1 - (b/Vm)} = \frac{1}{1 - \left(\frac{0.138}{35}\right)} = 1.004$$

42. A

Sol. SO_2 is reducing while TeO_2 is an oxidizing agent.

43. C

Sol. PbI_2 is formed which is yellow in colour.

44. B

Sol. $Cu_2S + 2KMnO_4 + 4H_2SO_4 \longrightarrow 2CuSO_4 + 2MnSO_4 + K_2SO_4 + 4H_2O$

45. C

Sol. At constant P and n $V \propto T$.

46. B

Sol. Greater the intermolecular attraction, lesser the volatility at a given temperature.

47. D

Sol. Overall reaction for electrolysis of K_2SO_4 is $2H_2O(\ell) \longrightarrow 2H_2 + O_2$

48. B

Sol. At equivalence point
Moles of $AgNO_3$ = Moles of KCl
$10^{-3} \times 20 \times M = 0.20 \times 20 \times 10^{-3}$
M = 0.20
Moles of $AgNO_3$ initially = M × V
$= \frac{0.20}{100} \times 20 \times 10^{-3}$
= 4×10^{-3}.

49. D

Sol. Chloride salts of mercury, silver and lead are not readily soluble in water.

50. C

Sol. Analgesics reduce or abolish pain without causing impairment of consciousness.

51. B

Sol. Protons in $NO_3^- = 31$

Moles of $NO_3^- = \frac{124}{62} = 2$ mole

1 mole $NO_3^- = 31\,N_A$ protons

2 mole $NO_3^- = 62\,N_A$

52. B

Sol. P(ideal) = 0.2 × 200 + 0.8 × 600
= 520 torr < P(obs)
Solution showing positive deviation.

53. A

Sol. $\Delta H_{reaction} = \sum \Delta H_f(\text{Products}) - \sum \Delta H_f(\text{Reactants})$
$\Delta H_f(B) - \Delta H_f(A) = 90$ kcal

$\Delta H_f(B) > \Delta H_f(A)$
$\Delta H_f(C) - \Delta H_f(B) = -70$ kcal
$\Delta H_f(B) > \Delta H_f(C)$
$A \longrightarrow C \qquad \Delta H = 20$ kcal
$\Delta H_f(C) > \Delta H_f(A)$
A < C < B

54. C

Sol. Density $= \frac{4 \times M}{N_A \times a^3}$

$$\frac{d_{NaCl}}{d_{KCl}} = \frac{58.5}{74.5}\left(\frac{a_{KCl}}{a_{NaCl}}\right)^3$$

$$d_{KCl} = 1.8 \times \frac{74.5}{58.5}\left[\frac{1.5\,r(Cl^-)}{1.8\,r(Cl^-)}\right]^3 = 1.33 \text{ g / cc.}$$

55. B

Sol. NaX = i = 2
$\Delta T_f = i \times K_f \times m$

$$1.27 = 2 \times 1.86 \times \frac{2}{M} \times \frac{1000}{100}$$

M = 58.5 = Molar mass of NaCl

56. D

Sol. Magnitude of W_{max} = nEF = 2 × 3.5 × 96500 J
= 6.75 × 10^5 J.
= 6.75 × 10^2 kJ.

57. C

Sol. The maximum limit of nitrate in drinking water is 50 ppm.
The prescribe upper limit of lead in drinking water is 50 ppb.

58. C

Sol. For Step-II

$$P_3 = \left(\frac{T_2}{T_3}\right)^{\frac{\gamma}{1-\gamma}} P_2 = \left(\frac{600}{300}\right)^{-\frac{7}{2}} \times 2$$

$$= 2^{-\frac{7}{2}} \times 2 = \frac{1}{4\sqrt{2}} \text{atm}$$

$$V_3 = \frac{RT_3}{P_3} = \frac{0.082 \times 300}{\frac{1}{4\sqrt{2}}} = 139 \text{ L.}$$

59. B

Sol. $\underset{}{FeO} + \underset{(\text{silica})}{SiO_2} + \underset{(\text{slag})}{FeSiO_3}$

60. D

Sol. If all the oleum is SO_3 then maximum weight of H_2SO_4 will be

$$100 + 100 \times \frac{18}{80} = 122.5$$

So % oleum cannot exceed 122.5%.

Mathematics

PART – III

SECTION – A

61. B

Sol. Roots of $x^3 - 9x^2 + ax - 24 = 0$ are in A.P.
$\Rightarrow 3c = 9 \Rightarrow c = 3$
$\Rightarrow a = 2,\ d = 4$
$\Rightarrow$ 2, 3, 4 also roots of equation $5x^4 + px^3 + qx^2 + rx + s = 0$
$\Rightarrow$ 2, b, 3, 4 are in H.P.
$\Rightarrow b = \frac{12}{5}$
$$\Rightarrow \left|\frac{P(x)}{Q(x)}\right| = \left|\frac{5(x-2)(x-3)(x-4)\left(x-\frac{12}{5}\right)}{1\cdot(x-2)(x-3)(x-4)}\right| = |(5x-12)|$$

62. C

Sol. $${}^nC_1 + {}^nC_2\cdot\alpha + {}^nC_3\cdot\alpha^2 + \ldots.. + {}^nC_n\cdot\alpha^{n-1}$$
$$= \frac{1}{\alpha}\left[{}^nC_1\cdot\alpha + {}^nC_2\cdot\alpha^2 + \ldots.. + {}^nC_n\cdot\alpha^{n-1}\right]$$
$$= \frac{1}{\alpha}\left[(1+\alpha)^n - 1\right]$$

63. C

Sol. Given numbers are 1, 2, 3, 4, ..., 2n + 1
Mean of these numbers = $\bar{x} = \frac{1+2+3+\ldots..+2n+1}{2n+1} = n+1$
$$\sigma^2 = \frac{1}{2n+1}\sum_{r=0}^{2n}\{(1+r)-(1+n)\}^2 = \frac{1}{2n+1}\sum_{r=0}^{2n}(n-r)^2 = \frac{2\left(1^2+2^2+\ldots..+n^2\right)}{2n+1}$$
$$\sigma^2 = \frac{n(n+1)}{3} \Rightarrow \sigma = \sqrt{\frac{n(n+1)}{3}}$$

64. D

Sol. Let $y = [x] \Rightarrow y^2 + ay + b = 0$
Let $a = 2m + 1,\ b = 2n + 1$
$\Rightarrow \Delta = a^2 - 4b = (2m+1)^2 - 4(2n+1) = 8K + 5$
If $8K + 5 = (2P+1)^2$ {where $P \in I$}
$\Rightarrow 4P^2 + 4P = 4(2K+1)$
$\Rightarrow P(P+1) = 2K + 1$ which is not possible
So, [x] = irrational $\Rightarrow x \in \phi$

65. D

Sol. $\dfrac{a^{n-\frac{1}{2}}+b^{n-\frac{1}{2}}}{a^{n+\frac{1}{2}}+b^{n+\frac{1}{2}}}=\dfrac{1}{\sqrt{ab}}$

$\Rightarrow \left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\left(a^{n}-b^{n}\right)=0 \Rightarrow n = 0 \text{ as } n \neq b$

66. B

Sol. $P(A)=\dfrac{33}{100}$, $P(B)=\dfrac{50}{100}$, $P(A\cap B)=\dfrac{16}{100}$

$P(A\cup B)=\dfrac{33+50-16}{100}=\dfrac{67}{100}$

67. D

Sol. $\left(e^{z^2}\right)=e^{\left(x^2-y^2\right)+2ixy}$

$\text{amp}\left(e^{z^2}\right)=2xy$

Similarly $\text{amp}\left(e^{(z+i)}\right)=(y+1)$

$2xy = y + 1 \Rightarrow y=\dfrac{1}{(2x-1)}$

$f(3)=\dfrac{1}{5}$

68. A

Sol. Take $z_1=1+i\sqrt{3}$ and $z_2 = 3$

$\left|\dfrac{z_1+z_2}{z_1-z_2}\right| = \sqrt{\dfrac{19}{7}}$

69. B

Sol. Put $\log_{\sqrt{3}}\tan x=t,\ t<0$

$t\sqrt{2t+3}=-1 \Rightarrow 2t^3 + 3t^2 - 1 = 0 \Rightarrow t = -1$

$\tan x=\dfrac{1}{\sqrt{3}} \Rightarrow x=\dfrac{\pi}{6}$ or $\dfrac{7\pi}{6}$

70. D

Sol. We require $t_n = t_m \Rightarrow 3n - 2 = 5m + 4,\ m, n \in N$,

$3n = 5m + 6 \Rightarrow \dfrac{m}{3}=\dfrac{n-2}{5} = k$ (where $k \in I$)

as $m, n \le 500 \Rightarrow k \le 99$

$(m, n) = (3, 7)$ or $(6, 12)$ or $(9, 17)$ or .....

71. A

Sol. Required probability = $\frac{^{20}C_1 \cdot {^{20}C_1}}{^{40}C_2} \cdot \frac{^{19}C_1 \cdot {^{19}C_1}}{^{38}C_2} \cdot \frac{^{18}C_1 \cdot {^{18}C_1}}{^{36}C_2} ..... \frac{^{1}C_1 \cdot {^{1}C_1}}{^{2}C_2}$ = $\frac{2^{20} \times (20!)^2}{40!}$

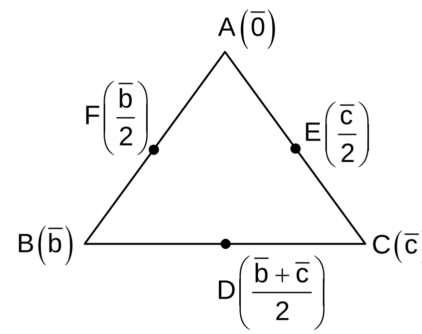
72. A

Sol. Consider A as origin LHS $\overrightarrow{AD} + \frac{2}{3}\overrightarrow{BE} + \frac{1}{3}\overrightarrow{CF}$

$\left(\frac{\bar{b}+\bar{c}}{2} - 0\right) + \frac{2}{3}\left(\frac{\bar{c}}{2} - \bar{b}\right) + \frac{1}{3}\left(\frac{\bar{b}}{2} - \bar{c}\right)$

= $\bar{b}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{6}\right) + \bar{k}\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{3}\right)$ = $0\bar{b} + \frac{1}{2}\bar{c}$

= $\frac{1}{2}\bar{A}C \therefore k = \frac{1}{2} \Rightarrow 2K = 1$



73. A

Sol. We know that $A(\text{adj } A) = |A|I$

A(Adj A) = 4I ⇒ |A| = 4

Again we know that $|\text{Adj } A| = |A|^{n-1}$

$\frac{|\text{Adj }(\text{Adj } A)|}{|\text{Adj } A|} = \frac{|A|^4}{|A|^2} = |A|^2 = 4^2 = 16$

74. C

Sol. The plane containing the given line is (2x + 3y + 5z + 1) + λ(3x + 4y + 6z + 2) = 0

∵ The plane is parallel to y-axis

$\Rightarrow \lambda = -\frac{3}{4}$

⇒ A point on y-axis is the origin and the perpendicular distance from the origin to the plane x – 2z + 2 = 0 is $\frac{2}{\sqrt{5}}$

75. C

Sol. cos α + i sin α is a root of $a_n\left(\frac{1}{z}\right)^n + a_{n-1}\left(\frac{1}{z}\right)^{n-1} + ... + a_2\left(\frac{1}{z}\right)^2 + a_1\left(\frac{1}{z}\right) + 1 = 0$. Equating real parts on both sides, $a_n \cos n\alpha + a_{n-1} \cos (n-1)\alpha + ... + a_1 \cos \alpha + 1 = 0$

76. B

Sol. AB = BA

$\Rightarrow a_{21} = 2a_{12}$ ..... (1)

$a_{11} = a_{22}$ ..... (2)

$|A| = 0 \Rightarrow a_{11} a_{22} = a_{21} \cdot a_{12}$ ..... (3)

From equation (1), (2) and (3), we get $\left(\frac{a_{11}}{a_{12}}\right)^2 = 2$

77. B

Sol. $z_1\left(z_1^2 - 3z_2^2\right) = 2.....(1)$; $z_2\left(3z_1^2 - z_2^2\right) = 11....(2)$ multiplying (2) by i add to (1) which gives $\left(z_1 + iz_2\right)^3 = 2 + 11i.....(3)$ and multiplying (2) by i and subtracting from (1) gives
$\left(z_1 - iz_2\right)^3 = 2 - 11i....(4)$
Now multiply (3) and (4) then $z_1^2 + z_2^2 = 5$

78. B

Sol. Since |z – 1| = 1 $\Rightarrow$ z – 1 = cis $\theta \Rightarrow$ $z = (1+\cos\theta) + i\sin\theta = 2\cos\frac{\theta}{2}\text{cis}\frac{\theta}{2}$
$\therefore\ \frac{1}{z} - \frac{1}{2} = \frac{\text{cis}\left(-\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}} - \frac{1}{2} = -\frac{i}{2}\tan\frac{\theta}{2}$ which is purely imaginary

79. C

Sol. $\vec{r} = \left(\vec{a}\times\vec{b}\right)\sin x + \left(\vec{b}\times\vec{c}\right)\cos y + 2\left(\vec{c}\times\vec{a}\right)$
$\vec{r}\cdot\left(\vec{a}+\vec{b}+\vec{c}\right) = 0$
$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]$(sin x + cos y + 2) = 0
$\left[\vec{a}\ \vec{b}\ \vec{c}\right] \neq 0 \Rightarrow$ sin x + cos y = –2
this is possible only when sin x = – 1 and cos y = –1
for $x^2 + y^2$ to be minimum x = $-\frac{\pi}{2}$ and y = π
$\Rightarrow$ Minimum value of $(x^2 + y^2)$ is = $\frac{\pi^2}{4} + \pi^2 = \frac{5\pi^2}{4}$

80. D

Sol. Since point of intersection of the given lines is (0, 0, 0). It must lie on the angle bisector so
$\frac{0-2}{-1} = \frac{0+2}{1} = \frac{0+k}{4} \quad \Rightarrow k = 8$

81. A

Sol. $(x + 1)(2x + 1)(2^2x + 1)(2^3x + 1) ...... (2^{20}x + 1)$
$= 1\cdot 2\cdot 2^2\cdot 2^3......2^{20}(x+1)\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2^2}\right)\left(x+\frac{1}{2^3}\right)........\left(x+\frac{1}{2^{20}}\right)$
$= 2^{\frac{20\times 21}{2}}(x+1)\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2^2}\right)........\left(x+\frac{1}{2^{20}}\right)$

Coefficient of x^{20} = $2^{210}\left(1+\frac{1}{2}+\frac{1}{2^2}+.........+\frac{1}{2^{20}}\right)$

= $\left(2^{21}\right)^{10}\left(\frac{1-\frac{1}{2^{21}}}{1-\frac{1}{2}}\right)$ = $(2^{21})^9\,2(2^{21}-1) = 2^{211} - 2^{190}$

82. A

Sol. For x axis $\vec{r} = a_1\hat{i} \Rightarrow \vec{n}\cdot\hat{i} = \frac{q}{a_1}$

$\vec{n}\cdot\hat{j} = \frac{q}{a_2}$

$\vec{n}\cdot\hat{k} = \frac{q}{a_3}$

$\vec{n} = \left(\vec{n}\cdot\hat{i}\right)\hat{i} + \left(\vec{n}\cdot\hat{j}\right)\hat{j} + \left(\vec{n}\cdot\hat{k}\right)\hat{k}$

$\vec{n} = \frac{q}{a_1}\hat{i} + \frac{q}{a_2}\hat{j} + \frac{q}{a_3}\hat{k}$

83. D

Sol. Probability that matrix is symmetric = $\frac{7^6}{7^9} = \frac{1}{7^3}$

Again that matrix is skew symmetric = $\frac{7^3}{7^9} = \frac{1}{7^6}$

One matrix containing all elements = 0; is common in both type of matrices

Required probability = $\frac{1}{7^3} + \frac{1}{7^6} - \frac{1}{7^9}$

84. C

Sol. $\because$ Plane contains the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$

a(x – 1) + b(y + 2) + cz = 0 ..... (1)

2a – 3b + 5c = 0 ..... (2)

a – b + c = 0 ..... (3)

Required plane is 2x + 3y + z + 4 = 0

85. C

Sol. $(abc + abd + acd + bcd)^{10}$

$= a^{10}b^{10}c^{10}d^{10}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$

$\therefore$ Coefficient of $a^8b^4c^9d^9$ = coefficient of $a^{-2}b^{-6}c^{-1}d^{-1}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$ = 2520

86. C

Sol. By Venn diagram $P\left(C\cap\left(\overline{A\cup B}\right)\right)=1-\left(\frac{1}{10}+\frac{1}{15}+\frac{1}{5}\right)=\frac{19}{30}$

87. D

Sol. abc = 1 in 1 ways
abc = 2, 3, 5, 7, 11 in 15 ways
abc = 4, 9 in 12 ways
abc = 8 in 10 ways
abc = 6, 10 in 18 ways
So, total number of solution is 56

88. B

Sol. Let the roots be $\alpha_1, \alpha_2, \ldots\ldots, \alpha_8$

$\Rightarrow \alpha_1+\alpha_2+\ldots\ldots+\alpha_8=4$, $\alpha_1\alpha_2\ldots\ldots\alpha_8=\frac{1}{2^8}$

$\Rightarrow (\alpha_1\alpha_2 \ldots\ldots \alpha_8)^{\frac{1}{8}}=\frac{1}{2}=\frac{\alpha_1+\alpha_2+\ldots\ldots+\alpha_8}{8}$

$\Rightarrow$ AM = GM $\Rightarrow$ all the roots are equal to $\frac{1}{2}$

$\Rightarrow a_1=-\,{}^8C_7\left(\frac{1}{2}\right)^7=-\frac{1}{2^4}$

$\Rightarrow a_2={}^8C_6\left(\frac{1}{2}\right)^6=\frac{7}{2^4}$

89. C

Sol. Let Z = a + ib, b $\neq$ 0 where Im Z = b
$Z^5=(a+ib)^5=a^5+{}^5C_1a^4bi+{}^5C_2a^3b^2i^2+{}^5C_3a^2b^3i^3+{}^5C_4ab^4i^4+i^5b^5$

$\text{Im } Z^5=5a^4b-10a^2b^3+b^5$

$y=\frac{\text{Im}Z^5}{\text{Im}^5 Z}=5\left(\frac{a}{b}\right)^4-10\left(\frac{a}{b}\right)^2+1$

Let $\left(\frac{a}{b}\right)^2=x(\text{say}),\ x\geq 0$

$y=5x^2-10x+1=5\left[x^2-2x\right]+1=5\left[(x-1)^2\right]-4$

Hence, $y_{min}=-4$

90. A

Sol. $x^2+1=(x+i)(x-i)$
b = 1, a = c
Number of ways of choosing a, b, c = 10 = 10 $\times$ 1
$\therefore$ K = 1