FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Advanced)-2019

PAPER –2

TEST DATE: 16-12-2018

Time Allotted: 3 Hours **Maximum Marks: 240**

General Instructions:

- The test consists of total 60 questions.
- Each subject (PCM) has 20 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-C.**

  Section–A (01 – 08, 21 – 28, 41 – 48) *contains 24 multiple choice questions which have one or more correct answer. Each question carries* ***+4 marks*** *for correct answer and* ***–2 marks*** *for wrong answer.*

  Section–A (09 – 12, 29 – 32, 49 – 52) *contains 12 paragraphs with each having 2 questions with one or more than one correct answer. Each question carries* ***+4 marks*** *for correct answer and* ***–2 marks*** *for wrong answer.*

  Section–C (13 – 20, 33 – 40, 53 – 60) *contains 24 Numerical based questions with answers as numerical value from 0 to 9 and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

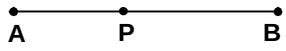
Physics

PART – I

SECTION – A
(One OR More Than One Choice Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **one or more than one** is/are correct

1. A particle of mass m = 3kg is constrained to move along a straight line. A and B are two fixed points on the line at a separation of L = ℓ m. When the particle is at some point P between A and B, it is acted upon by two forces $\vec{F}_1 = (200\ N/m)\overrightarrow{PA}$ and $\vec{F}_2 = (100\ N/m)\overrightarrow{PB}$, where magnitudes of $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are in metre. At time t = 0, the particle is projected from A towards B with speed v = $\frac{10}{\sqrt{3}}$ m/s. Then



(A) Particle reaches to B at t = $\frac{\pi}{30}$ sec

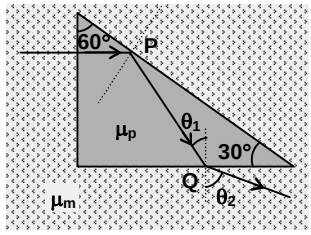
(B) Particle reaches to B at t = $\frac{\pi}{15}$ sec

(C) Oscillation energy of the particle is $\frac{200}{3}$ J

(D) Oscillation energy of the particle is $\frac{100}{3}$ J

2. In the Bohr's hydrogen atom model, R, V and E represent the radius of the orbit, speed of the electron and total energy of the electron respectively. Which of the following quantities are proportional to the quantum number n?
(A) VR
(B) RE
(C) VE^{-1}
(D) RE^{-1}

3. A ray of light is incident normally on one face 30° – 60° – 90° prism of refractive index μ_p = 5/3 immersed in water of refractive index μ_m = 4/3 as shown in the figure.



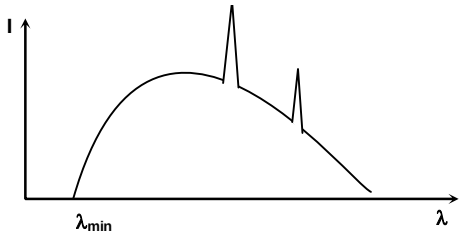
(A) The exit angle θ_2 of the ray is $\sin^{-1}\left(\frac{5}{8}\right)$

(B) The exit angle θ_2 of the ray is $\sin^{-1}\left(\frac{5}{4\sqrt{3}}\right)$

(C) Total internal reflection at point P ceases if the refractive index of water is increased to $n > \frac{5}{2\sqrt{3}}$ by dissolving some substance.

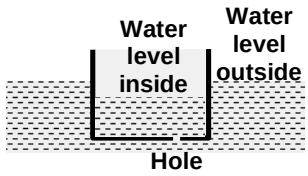
(D) Total internal reflection at point P ceases if the refractive index of water is increased to $n > \frac{5}{6}$ by dissolving some substance.

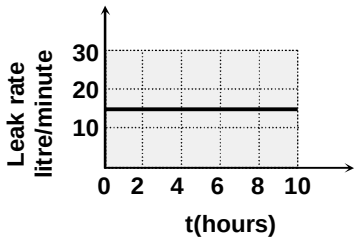
4. A beam of electrons striking a copper target produces X-rays. Its spectrum is as shown in the figure. Keeping the voltage same if the copper target is replaced with a different metal, the cut off wavelength and characteristic lines of the new spectrum will change in comparison with old as



(A) Cut off wavelength will remain unchanged.
(B) Both cut off wavelength and characteristic lines must remain unchanged.
(C) Characteristic lines may be different.
(D) Cut off wavelength will be different while characteristic lines may remain unchanged.

5. A small hole is punched into the bottom of a rectangular boat, allowing water to enter the boat. As the boat sink into the water, which of the following graph best shows how the water flow rate through the hole varies with time? Assume that the boat remains horizontal as it sinks.



(A)


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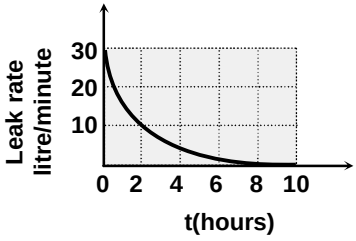

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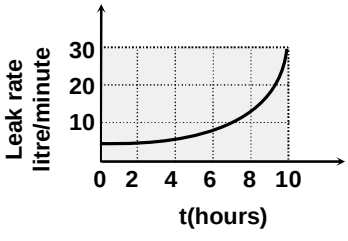
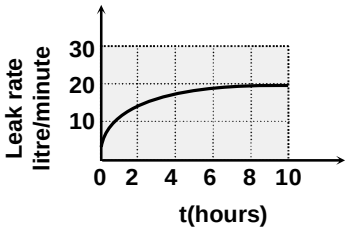


(D)


6. A very large number of small particles forms a spherical cloud. Initially they are at rest, have uniform mass density per unit volume ρ_0 and occupy a region of radius r_0. The cloud collapses due to gravitation, the particles do not interact with each other in any other way. How much time passes until the cloud collapse fully?

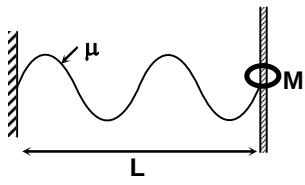
(A) $\sqrt{\frac{3\pi}{8G\rho_0}}$

(B) $\sqrt{\frac{\pi}{32G\rho_0}}$

(C) $\sqrt{\frac{3\pi}{32G\rho_0}}$

(D) $\sqrt{\frac{\pi}{8G\rho_0}}$

7. Small amplitude standing waves of wavelength λ occur on a string with tension, T, mass per unit length μ and length L. One end of the string is fixed and the other end is attached to a ring of mass M that slides on a frictionless rod, as shown in the figure above. When gravity is neglected, which of the following conditions correctly determines the wavelength?



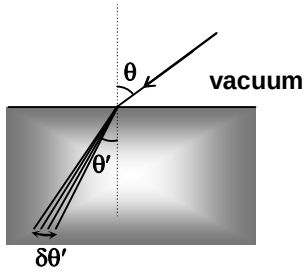
(A) $\frac{\mu}{M}=\frac{2\pi}{\lambda}\cot\left(\frac{2\pi L}{\lambda}\right)$

(B) $\frac{\mu}{M}=\frac{2\pi}{\lambda}\tan\left(\frac{2\pi L}{\lambda}\right)$

(C) $\lambda=\frac{2L}{n},\ n=1,2,3,.....$

(D) $\lambda=\frac{2L}{\left(n+\frac{1}{2}\right)},\ n=1,2,3,.....$

8. A beam of light has a small wavelength spread $\delta\lambda$ above a central wavelength λ. The beam travels in vacuum until it enters a glass plate at an angle θ relative to the normal to the plate as shown in the figure. The index of refraction of the glass is given by $n(\lambda)$. The angular spread $\delta\theta'$ of the refracted beam is given by



(A) $\delta\theta'=\left|\frac{d\mu(\lambda)}{d\lambda}\delta\lambda\right|$

(B) $\delta\theta'=\left|\frac{\tan\theta'}{n}\frac{dn(\lambda)}{d\lambda}\delta\lambda\right|$

(C) $\delta\theta'=\left|\frac{\sin\theta}{\sin\theta'}\frac{\delta\lambda}{\lambda}\right|$

(D) $\delta\theta'$ remains constant for different values of θ in the range $\frac{\pi}{6}\le\theta\le\frac{\pi}{3}$.

Comprehension Type

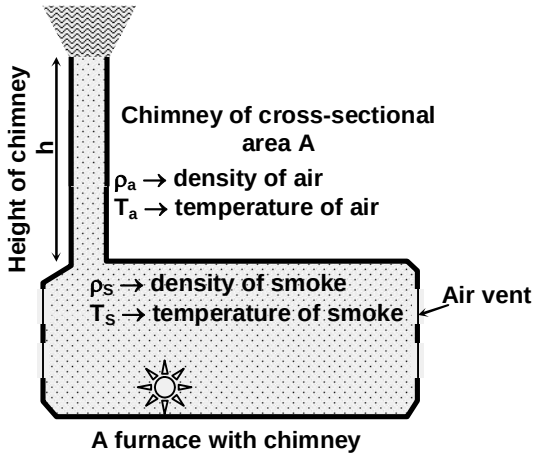
This section contains **2 paragraphs** each describing theory, experiment, data etc. Each question has **four** options (A), (B), (C) and (D). **One or more than one** of these four option(s) is(are) correct

Paragraph for Questions 09 & 10

Gaseous products of burning are released into the atmosphere of temperature T_a through a high chimney of cross section A and height h as shown in the figure. The solid matter is burnt in the furnace which is at temperature T_S. due to burning smoke (gas) generation rate (volume per unit time) is Q.

It can be assume that

- → the velocity of the smoke (gas) in the furnace is negligibly small.
- → the density of the smoke (gas) does not differ from that air at the same temperature and pressure.
- → the gases can be treated as ideal while in furnace.
- → the pressure of the air changes with height according to the hydrostatic law. The change of the density of the air with height is negligible.
- → the flow of gases in the chimney follows Bernoulli's equation.
- → the change of the density of the gas (smoke) is negligible throughout the chimney.



A furnace with chimney

9. What is the minimum height of the chimney needed in order that chimney functions efficiently, so that it can release all of the produced smoke(gas) into the atmosphere? Express your answer in terms of Q, A, g, T_a, ρ_a, ρ_s and ΔT, where $\Delta T = T_S - T_a$

(A) $\dfrac{\rho_s T_a Q^2}{gA^2\Delta T(\rho_a - \rho_s)}$

(B) $\dfrac{T_a Q^2}{2gA^2\Delta T}$

(C) $\dfrac{Q^2\rho_s}{2(\rho_a - \rho_s)A^2 g}$

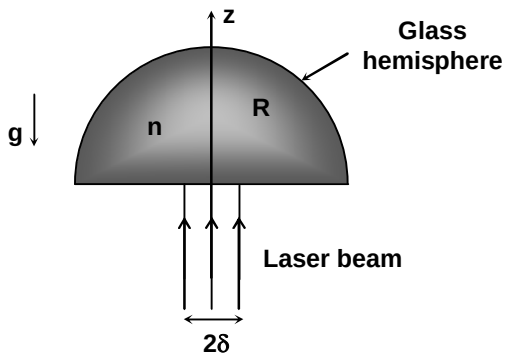
(D) $\dfrac{Q^2(T_a + \Delta T)}{2gA^2\Delta T}$

10. Assume that two chimneys are built to serve exactly the same purpose. Their cross sections are identical, but are designed to work in different parts of the world, one in cold regions designed to work at an average atmospheric temperature of $-23°C$ and the other in warm regions, designed to work at an average atmospheric temperature of 27°C. The temperature of the furnace is 327°C. It was calculated that the height of the chimney designed to work in cold regions is 100 m. How high is the other chimney?

(A) 71.4m
(B) 280 m
(C) 140 m
(D) 100 m

Paragraph for Questions 11 & 12

In a hypothetical experiment light is used to levitate a transparent glass hemisphere with radius R and mass m having refractive index n. In the medium outside the hemisphere, the index of refraction is equal to one. A parallel beam of monochromatic laser light is incident uniformly and normally onto the central portion of its planar surface as shown in figure. The acceleration of gravity g is vertically downwards. The radius δ of the circular cross section of the laser beam is much smaller than R. Both the glass hemisphere and the laser beam are axially symmetric with respect to z-axis.



The glass hemisphere does not absorb any laser light. Its surface has been coated with a thin layer of transparent material so that reflections are negligible when light enters and leaves the glass hemisphere. The optical path traversed by laser light passing through the non-reflecting surface layer is also negligible.

11. Find the laser power P required to balance the weight of the glass hemisphere.

(A) $\dfrac{2mgR^2c}{(n-1)^2\delta^2}$

(B) $\dfrac{4mgR^2c}{(n-1)^2\delta^2}$

(C) $\dfrac{2mgR^2c}{(n^2-1)\delta^2}$

(D) $\dfrac{mgR^2c}{(n^2-1)\delta^2}$

12. If the frequency of the laser beam is f, then the number of photons striking the hemisphere per unit time is

(A) $\dfrac{2mgR^2c}{(n-1)^2\delta^2hf}$

(B) $\dfrac{4mgR^2c}{(n-1)^2\delta^2hf}$

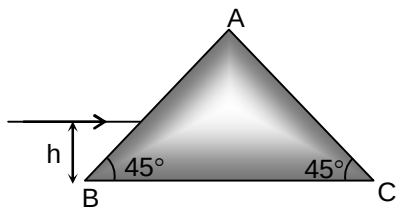
(C) $\dfrac{2mgR^2c}{(n^2-1)\delta^2hf}$

(D) $\dfrac{mgR^2c}{(n^2-1)\delta^2hf}$

SECTION – C
(One Integer Value Correct Type)

This section contains **8 questions**. Each question, when worked out will result in **one integer** from 0 to 9 (both inclusive).

13. A ray of light is incident parallel to BC at a height h = 3.0 cm from BC. Find the height (in cm) above BC at which the emergent ray leaves the surface AC. It is given that $\mu = \sqrt{2}$ and length BC = 20 cm. Take tan 15° = 0.25.

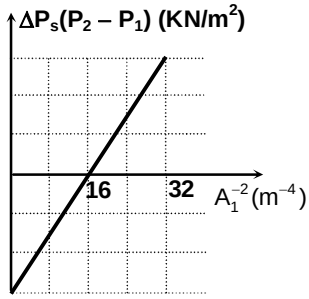


14. A compound microscope is used to enlarge an object kept at a distance of 3 cm from its objective. The objective consists of several convex lenses is contact and has a focal length of 2 cm. If a lens of focal length 10 cm is removed from the objective, the eyepiece has to be moved by x cm to refocus the image. The value of x is

15. Consider a nuclear reaction A + B $\rightarrow$ C. A nucleus A moving with kinetic energy of 5 MeV collides with a nucleus B moving with kinetic energy of 3 MeV and forms a nucleus C in excited state. Find the kinetic energy of nucleus C (in MeV) just after its formation if it is formed in a state with excitation energy 10.65 MeV. Take masses of nuclei of A, B and C as 25.0, 10.0 and 34.995 amu, 1 amu $\approx$ 930 MeV/c^2

16. A small sphere of radius R = 10 cm is arranged to pulsate so that its radius varies in simple harmonic motion between a range 10.0cm $\pm$ 0.1cm with frequency f = 100 Hz. This produces sound waves in the surrounding air of density ρ =1.6 kg/m^3 and bulk modulus B = 10^5 Pascal. Find the total acoustic power radiated by the sphere (in watt). (take $\pi^3 = \frac{225}{8}$)

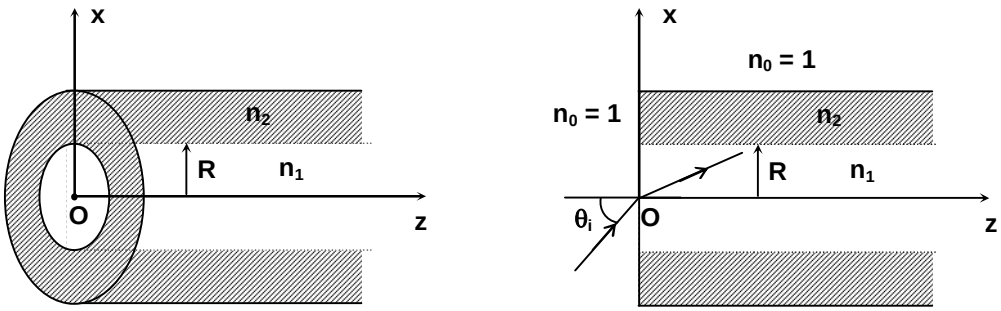
17. Fresh water flows horizontally from pipe section 1 of cross sectional area A_1 into pipe section 2 of cross-sectional area A_2. Figure gives a plot of the pressure difference ($P_2 - P_1$) versus the inverse area squared A_1^{-2} that would be expected for a volume flow rate of a certain value if the waver flow were laminar under all circumstances. The scale on the vertical axis is set by ΔP_S = 100 KN/m^2. For the condition of the figure, what is the volume flow rate in m^3/s? (take $\sqrt{6} = 2.4$)



18. A plano-convex lens is placed on a plane glass surface. A thin film of air is formed between the curved surface of the lens and the plane glass plate. The thickness of the air film is zero at the point of contact and increases as one moves away from the point of contact. In the arrangement light containing two wavelengths 4000 Å and 4002 Å is allowed to fall normally on the flat face of the lens. Calculate the minimum distance (in cm) from the point of contact at which the rings will disappear. Assume that the radius of curvature of the curved surface is 400 cm. (assume identical intensities for both wavelengths)

19. An optical fiber consists of a cylindrical core of radius R, made of transparent material with refraction index varying gradually from the value n = n_1 on the axis to n = n_2 (with $1 < n_2 < n_1$) at a distance R from the axis, according to the formula

$$n = n(x) = n_1\sqrt{1-\alpha^2x^2}$$



Where x is the distance from the core axis and α is a constant. The core is surrounded by a cladding made of a material with constant refraction index n_2. Outside the fibre is air of refractive index n_0. Let Oz be the axis of the fiber, with O – the centre of the fiber end. Given n_0 = 1, n_1 = 1.5 and n_2 = 1.46, R = 25 μm. A monochromatic light ray enters the fiber at point O under an incident

angle $\theta_i = 30°$, the incident plane being the plane xOz. At each point on the trajectory of the light in the fiber, the refractive index n and the angle θ between the light ray and the Oz axis satisfy the relationship $n\cos\theta = \sqrt{C}$. Find C.

20. Two thin lenses with lens powers D_1 and D_2 diopter are located at distance L = 25 cm from each other and their optical axes coincide. This system creates an erect real image of the object, located at the optical axis closer to lens D_1 with the net magnification $M_1 = 1$. If the position of the two tenses are exchanged, the system again produces an erect real image with the net magnification $M_2 = 4$. What is the difference between lens powers $\Delta D = D_1 - D_2$ (in diopter)?

Chemistry

PART – II

SECTION – A
(One OR More Than One Choice Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **one or more than one** is/are correct

21. Which of the following statement(s) is/are correct?
    (A) AgBr shows both Frenkel and Schottky defect.
    (B) When crystals of NaCl are heated in an atmosphere of Na vapours. Crystal shows yellow colour due to metal excess defect.
    (C) ZnO on heating turns yellow this is because of metal excess defect due to the presence of extra cations at interstitial sites.
    (D) Schottky defect is shown by ionic compounds in which cation and anion are of almost similar sizes.

22. Which of the following statement(s) is/are correct?
    (A) Melting point order: $PH_3 < AsH_3 < SbH_3 < NH_3$
    (B) Boiling point order: $PH_3 < AsH_3 < NH_3 < SbH_3$
    (C) NCl_3 on hydrolysis give HOCl while PCl_3 gives HCl
    (D) Melting point order: HCl < HBr < HF < HI

23. Molarity of solution, prepared by mixing CaO in 200 ml water. The CaO is obtained by heating 100 gm of limestone which was 80% pure?
    (A) 5
    (B) 4
    (C) 3
    (D) 2

24. Which of the following statement(s) is/are correct?
    (A) Fe^{3+} ion forms a charge transfer complex with SCN^- ions.
    (B) When KCN is added to $CuSO_4$ solution, KCN acts as a reducing agent as well as a complexing agent.
    (C) ZnO is reduced to Zn by H_2.
    (D) In Baeyer's process Al_2O_3 goes in to $Al(OH)_4^-$ while other basic oxides remains insoluble.

25. Correct order among the following is/are?
    (A) HF > HCl > HBr > HI (Dipole moment)
    (B) $HOCl < HClO_2 < HClO_3 < HClO_4$ (Acidic strength)
    (C) $H_2S < H_2Se < H_2Te < H_2O$ (Boiling point)
    (D) $NH_3 > PH_3 > AsH_3 > SbH_3$ (Bond angle)

26. The E^o value of the following reaction are given

$Sn^{2+} \longrightarrow Sn^{4+} + 2e^-$ $\quad E^o = -0.15$ V

$Fe \longrightarrow Fe^{2+} + 2e^-$ $\quad E^o = +0.44$ V

$Fe^{2+} \longrightarrow Fe^{3+} + e^-$ $\quad E^o = -0.771$ V

$Fe \longrightarrow Fe^{3+} + 3e^-$ $\quad E^o = -0.036$ V

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$ $\quad E^o = +1.51$ V

Under standard condition which of the following reaction is spontaneous?

(A) $Fe + MnO_4^- \xrightarrow{H^+} Fe^{2+} + Mn^{2+} + H_2O$

(B) $Fe^{2+} + MnO_4^- \xrightarrow{H^+} Fe^{3+} + Mn^{2+} + H_2O$

(C) $Fe + MnO_4^- \xrightarrow{H^+} Fe^{3+} + Mn^{2+} + H_2O$

(D) $Fe^{2+} + Sn^{4+} \longrightarrow Fe^{3+} + Sn^{2+}$

27. When one mole of triatomic (bent) ideal gas at 300 K undergoes reversible adiabatic change under a constant external pressure of 1 atm from 1 lit volume to 27 litre. What would be the final temperature (in Kelvin)? [$\gamma = 1 + \frac{2}{f}$: where f is degree of freedom].

(A) 100
(B) 80.27
(C) 11.11
(D) 33.33

28. 1 mole of an ideal gas at 300 K expands isothermally and reversibly ten times of its initial volume. Energy used in non useful work is

(A) 5.744 kJ/mol
(B) 0.0191 kJ/mol
(C) 2.303 nRT log V_2/V_1
(D) 2.303 nR log V_2/V_1

Comprehension Type

This section contains **2 paragraphs** each describing theory, experiment, data etc. Each question has **four** options (A), (B), (C) and (D). **One or more than one** of these four option(s) is(are) correct

Paragraph for Question Nos. 29 to 30

$$Mn(NO_3)_2 \xrightarrow{\Delta} \underset{(Brown)}{A(s)} + B(g)\uparrow$$

$$Conc.\ H_2SO_4 + A \longrightarrow D(aq.) + E(g) + H_2O$$

$$D + NaOH \longrightarrow \underset{White\ ppt.}{F} \xrightarrow{E} (Brown)$$

29. $D + PbO_2 + Conc.\ HNO_3 \longrightarrow \underset{(Purple)}{Y}$

Y is:

(A) H_2MnO_4
(B) $HMnO_4$
(C) Pb_3O_4
(D) $Mn(NO_3)_2$

30. Which of the following is/are correct?

(A) $D + NaBiO_3 + dil.\ HNO_3 \longrightarrow$ Purple colour

(B) $D + IO_4^- + H_2O \xrightarrow{H^+}$ Purple colour

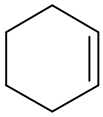
(C) $B + \underset{(Excess)}{FeSO_4} \longrightarrow$ Brown ring

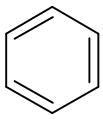
(D) $B + KI \xrightarrow{Starch\ paper}$ Blue spot

Paragraph for Question Nos. 31 to 32

Resonance energy of A molecule is equal to the difference between potential energy of most stable resonating structure and the hybrid (actual) structure. More the magnitude of resonance energy, more will be the stability of the molecule.

31. Calculate the magnitude of resonance energy of benzene

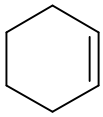
Given : $\Delta H_{Hydrogenation}$ of  = - 28.6 kcal/mole

$\Delta H_{Hydrogenation}$ of  = - 49.8 kcal/mole

(A) 35
(B) 36
(C) 70
(D) 72

32. Which of the following statement is/are correct?
Given: $\Delta H_{hydrogenation}$ of benzene $= -49.8$ kcal / mole

$\Delta H_{hydrogenation}$ of naphthalene $= -82$ kcal / mole

$\Delta H_{hydrogenation}$ of  $= -28.6$ kcal / mole

(A) Magnitude of resonance energy of naphthalene is = 61 k cal/mole
(B) Thermodynamically benzene is more stable than naphthalene
(C) Thermodynamically naphthalene is more stable than benzene
(D) Magnitude of resonance energy of naphthalene is 72 kcal/mole

SECTION – C
(One Integer Value Correct Type)

This section contains **8 questions**. Each question, when worked out will result in **one integer** from 0 to 9 (both inclusive).

33. Oxidation number of Cr in K_3CrO_8 is:

34. Coordination number of cation present in a void when r^+ / r^- lies in between 0.155 to 0.225.

35. A 6 ampere current is passed through a solution of $CuSO_4$ for 59 minutes. The amount of Cu (in grams) deposited at cathode is:

36. What is the normality of 28 V H_2O_2 solution ?

37. What is the percentage of SO_3 by mass in 101.8% oleum?

38. The n-factor of Br_2 in following reaction is x

$Br_2 + KOH \xrightarrow{hot} Br^- + BrO_3^-$

Than 3x = y : value of y is:

39. At 90^oC vapour pressure of pure liquid A is 500 mm Hg and that of pure liquid B is 900 mm Hg. If a mixture of solution of A and B boils at 90^oC and 1 atm pressure mole fraction of A is X_A:
$X_A \times 20 = n$, value of n is

40. Mixture of Fe_2O_3 and Al is used in solid fuel rocket. What is the fuel value (magnitude only) per ml of mix (in Kcal/ml).
Given: $\Delta H_{Al_2O_3} = -399.0 \text{ Kcal/mol}, \Delta H_{Fe_2O_3} = -199.0 \text{ Kcal/mol}$
Density of Fe_2O_3 and Al are 5 gm/ml and 3 gm/ml respectively.

Mathematics

PART – III

SECTION – A
(One OR More Than One Choice Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **one or more than one** is/are correct

41. Let U_n be the set of the nth roots of unity. If $A = \prod_{\alpha \in U_n} \left(\alpha + \frac{1}{\alpha} \right)$, then which of following is/are correct?

(A) $A = 0$, if $n = 4k$, $k \in N$
(B) $A = -4$, if $n = 4k + 2$, $k \in N$
(C) $A = 2$ if $n = 4k + 1$, $k \in N$
(D) $A = 2$, if $n = 4k + 3$, $k \in N$

42. Let $R_n = \left| b_{ij} \right|_{n \times n}$ be a determinant such that $b_{ij} = \begin{cases} 1 & , \quad |i-j| = 1 \\ 3 & , \quad |i-j| = 0 \\ 0 & , \quad \text{otherwise} \end{cases}$, then $n \in N$

(A) $R_{2019} + R_{2017} = 3R_{2018}$
(B) $R_{2019} + R_{2017} = 9R_{2018}$
(C) The value of R_4 is 54
(D) R_5 is multiple of 9

43. If P and Q are two 3×3 non-identity matrices with real entries such that $PP^T = I$, $QQ^T = I$ (P^T, Q^T are transpose of matrices P and Q respectively) and $\det(PQ) < 0$, then which of the following is/are always correct (where det stands for determinant of a matrix)

(A) $\det(P + Q) < 0$
(B) $\det(P + Q) = 0$
(C) atleast one of the $\det(P - I)$ or $\det(Q - I)$ will be zero
(D) $\det(P^{-1} + Q^{-1}) = \det(P + Q)$

44. Given in the Argand plane two curves $C_1 : \arg\left(\frac{z+1}{z-1} \right) = -\frac{\pi}{3}$ and $C_2 : \arg\left(\frac{z+1}{z-1} \right) = -\frac{2\pi}{3}$ where $z = x + iy$, $x, y \in R$, then

(A) sum of areas enclosed by the real axis and C_1, and the real axis and C_2 is equal to $\frac{4\pi^2}{3}$
(B) the number of complex numbers satisfying curve C_1 and C_2 simultaneously, is zero
(C) if α moves on curve C_2, then $\sqrt{\frac{1}{3}} \le |\alpha| < 1$, where $|\alpha|$ is modulus of α
(D) if β moves on curve C_1, then $|\beta|$ can be 1 unit, where $|\beta|$ is modulus of α

45. There are 6 red balls and 8 green balls in a bag, five balls are drawn out at random and placed in a red box and the remaining 9 balls are put in a green box. If the number of red balls in the green box is R_g and the number of green balls in the red box is G_r, then which of the following is/are **INCORRECT?**

(A) probability that $R_g + G_r$ is a prime number is $\frac{788}{1001}$
(B) probability that $R_g + G_r$ is a prime number is $\frac{213}{1001}$

(C) probability that $R_g + G_r$ is an even number is $\frac{708}{1001}$

(D) probability that $R_g + G_r$ is an odd number is $\frac{213}{1001}$

46. A five digit number $a_1a_2a_3a_4a_5$ is said to satisfy property P if $a_i \in \{1, 2, 3, 4, 5, 6, 7\}\ \forall\ 1 \le i \le 5$, and let $k = a_1 + a_2 + a_3 + a_4 + a_5$, then

(A) number of five digit numbers satisfying P such that k is divisible by 3 is $\left(\frac{7^5-1}{3}\right)$

(B) number of five digit numbers satisfying P such that k is divisible by 2 is $\left(\frac{7^5-1}{2}\right)$

(C) number of five digit numbers satisfying P such that k is divisible by 4 is $\left(\frac{7^5-3}{4}\right)$

(D) number of five digit numbers satisfying P such that that k = 33, is less than 20

47. An urn contains ω white balls ($\omega \ge 3$) and r red balls. If 3 balls were to the drawn without replacement, the probability that they would all be white is P. Introducing an extra white ball in the urn changed this probability to $\frac{4P}{3}$, then

(A) the maximum value of r can be 108
(B) the maximum value of r can be 88
(C) ω can be 6
(D) ω can be 9

48. Object A and B start moving simultaneously in the coordinate plane via a sequence of steps. each of length one with the same speed, object A starts at (0, 0) and each of its step is either right or up, both equally likely object B starts at (3, 5) and each of its steps is either left or down both equally likely, then which of the following is/are true

(A) the probability that object A and B meets is $\frac{5}{32}$

(B) the probability that object A and B will not meet is $\frac{25}{32}$

(C) the probability that A and B meet at point (a, b), such that a < b is $\frac{7}{64}$

(D) the probability that A and meet at point (a, b) where a = b is $\frac{3}{32}$

Comprehension Type

This section contains **2 paragraphs** each describing theory, experiment, data etc. Each question has **four** options (A), (B), (C) and (D). **One or more than one** of these four option(s) is(are) correct

Paragraph for Question Nos. 49 to 50

Read the following write up carefully and answer the following questions:

Let $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$; x, y, z $\in$ I with the condition that det(adj(adj A)) = $2^{12} \times 3^8 \times 5^4$, S_1 is set of all the matrices A, S_2 is set of all such matrices A where trace of A is divisible by 2, and x, y, z $\in$ N and S_3 is set

of all such matrices A, where (trace A^3) – 3 det A = 0, and x, y, z $\in$ N. Let the number of elements in set S_1, S_2 and S_3 be **2α, β** and **γ** respectively.

49. Then $\frac{\alpha+\gamma}{\beta}$ is divisible by

(A) 2 (B) 3
(C) 5 (D) 7

50. Which of the following is/are correct?

(A) $\frac{\alpha}{\beta+\gamma}<10$ (B) $\frac{\alpha}{\beta-\gamma}>15$

(C) $\frac{\alpha}{\beta+\gamma}>9$ (D) $\frac{\alpha}{\beta-\gamma}<14$

Paragraph for Question Nos. 51 to 52

Read the following write up carefully and answer the following questions:

Consider a tetrahedron Die that has four integers 1, 2, 3 and 4 written on its faces. Roll the Die 2000 times and for each i, $1 \le i \le 4$ let f(i) represent the number of times i is written on the bottom face.
Let A denote total sum of the numbers on the bottom face for these 2000 rolls, it A^4 = 6144 f(1) f(2) f(3) f(4), then

51. The value of $\left|\frac{f(1)-f(2)}{f(4)}\right|$ is equal to

(A) $\frac{f(2)}{f(4)}$

(B) $\frac{f(2)}{f(3)}$

(C) $\frac{f(1)}{f(2)}$

(D) none of these

52. Let g(x) is a polynomial equation such that, g(x) = 0, has all real roots, where
let $g(x) = x^4 + (f(1) - d)x^3 + (f(2) - c)x^2 + (f(3) - b)x + (f(4) - a)$, a, b, c, d $\in$ R |g(–i)| = 1, where $i = \sqrt{-1}$, then which of the following is/are **INCORRECT**?

(A) g′(5) > 4
(B) g(5) = 2000
(C) c + d < a + b
(D) g(4) > f(4)

SECTION – C
(One Integer Value Correct Type)

This section contains **8 questions**. Each question, when worked out will result in **one integer** from 0 to 9 (both inclusive).

53. Let $A=\begin{bmatrix}4 & 1\\ -9 & -2\end{bmatrix}$ and $A^{100}=\begin{bmatrix}a_1 & a_3\\ a_2 & a_4\end{bmatrix}$, then value of $a_1 + a_4$ is equal to _____

54. Let $f(x)=\begin{vmatrix} 7 & 2 & x^2-12 \\ 6 & x^2-12 & 3 \\ x^2-12 & 2 & 7 \end{vmatrix}$, then the number of real roots of f″(x) = 0 is equal to _____

55. For real a, b, c; a + b + c = 2, $a^2 + b^2 + c^2 = 6$ and $a^3 + b^3 + c^3 = 8$, then $\frac{(1-a)(1-b)(1-c)}{5}$ is equal to _____

56. If $R_n=\sqrt{1+\left(1-\frac{1}{n}\right)^2}+\sqrt{1+\left(1+\frac{1}{n}\right)^2}$, where $n \geq 1$, then $\sum_{n=1}^{20}\frac{1}{R_n}$ is equal to _____

57. The radius of circumscribing sphere of a regular tetrahedron the coordinates of whose vertices are A(0, 0, 0), B(3, 0, 0), C(0, 3, 0), D(0, 0, 3) is R, then [R], is equal to _____ (where [.] denotes the greatest integer function)

58. A variable line passing through the point of intersection of the lines $\frac{x-1}{7}=\frac{y-2}{1}=\frac{z+2}{-5}$ and x + 3 = y – 4 = –z. If S(x, y, z) = 0 is the locus of mirror image of the point (8, 3, –7) in the variable line and S′(x, y, z) = 0 is locus of foot of the perpendicular from (8, 3, –7) to the variable line, then the ratio of volumes of S to S′ is equal to _____

59. Let $P=\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$, where $a_i \in \{0, 1, 2\}\ \forall\ i \in \{1, 2, 3, ..., 9\}$. If possible values of det(P) are $P_1, P_2, P_3, ..., P_n$, then $P_1 + P_2 + P_3 + ... + P_n$ is equal to _____

60. Let R be the number of non-empty subsets of the set {1, 2, 3, ..., 9) satisfying the property that no two consecutive numbers are elements of a set, then number of prime divisors of R is/are_____