FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Advanced)-2019

PAPER –2

TEST DATE: 16-12-2018

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. B, C

Sol. $m\vec{a} = -200\vec{x} + 100(\vec{L} - \vec{x})$

$\therefore \quad a = -\frac{300}{3}\left(x - \frac{1}{3}\right)$

So, ω = 10 rad/s

For amplitude, $\frac{10}{\sqrt{3}} = 10\sqrt{A^2 - \frac{1}{9}}$

$\therefore \quad A = \frac{2}{3}m$

So, $t_{AB} = \frac{T}{12} + \frac{T}{4} = \frac{T}{3}$, where $T = \frac{2\pi}{10}$ sec

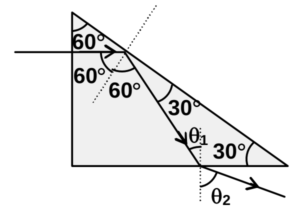
and $E_{osc} = \frac{1}{2}(3)(100)\left(\frac{4}{9}\right) = \frac{200}{3}J$

2. A, C

Sol. From Bohr's theory, $R \propto \frac{n^2}{z}$, $V \propto \frac{Z}{n}$ and $E \propto \frac{Z^2}{n^2}$

3. A, C

Sol. $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$



$\frac{5}{3}\sin 30° = \frac{4}{3}\sin\theta_2$

$\therefore \theta_2 = \sin^{-1}\left(\frac{5}{8}\right)$

for total internal reflection at P, we use

$\frac{5}{3}\sin 60° < n$

$n > \frac{5}{2\sqrt{3}}$

4. A, C

Sol. Cut off wavelength depends upon the accelerating potential difference so it will remain same and characteristic lines are dependent upon the energy gap between the energy levels of the elements. So these may change.

5. A

Sol. Difference in the outer and inner water levels always remains the same. Then the pressure difference across the hole remains same.

6. C

Sol. From Kepler's law, $T^2 = \frac{4\pi^2}{G\frac{4}{3}\pi r_0^3}\left(\frac{r_0}{2}\right)^3$

7. B

Sol. consider the case for $M \to 0$ and $M \to \infty$

8. B

Sol. (1) $\sin\theta = n(\lambda)\sin\theta'$

So, $0 = \sin\theta'\frac{dn(\lambda)}{d\lambda} + n(\lambda)\cos\theta'\frac{d\theta'}{d\lambda}$

$\therefore d\theta' = -\frac{\sin\theta'}{\cos\theta}\frac{1}{n(\lambda)}\frac{dn(\lambda)}{d\lambda}d\lambda$

$\therefore \delta\theta' = -\frac{\tan\theta'}{n(\lambda)}\frac{dn(\lambda)}{d\lambda}\delta\lambda$

9. B, C

10. C

Sol. (for Q. 9-10)

Applying Bernoulli's theorem between top and bottom of chimney

$P + \rho_s gh + \frac{1}{2}\rho_s v^2 = P_0$, where v = Q/A

So, $P = P_0 - \rho_s gh - \frac{1}{2}\rho_s\frac{Q^2}{A^2} = P_0 - \rho_a gh$

So, $h = \frac{\rho_s Q^2}{2(\rho_a - \rho_s)A^2 g}$

For ideal gas, $\frac{\rho_a}{\rho_s} = \frac{T_s}{T_a}$, So $h = \frac{T_a Q^2}{2gA^2\Delta T}$

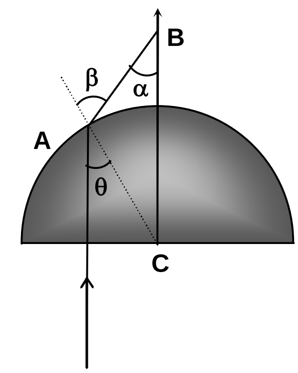
$\frac{h_W}{h_C} = \frac{(273+27)}{(327-27)}\frac{(327+23)}{(273-23)}$, where h_C = 100 m

So, h_w = 140 m

11. B

12. B

Sol. (for Q. 11 to 12)



For small θ, $n\theta = \beta$

So, $\alpha = (n-1)\theta$

Initial photon momentum per unit time = P/c

Final photon momentum per unit time

$= \int \frac{P}{\pi\delta^2}\frac{2\pi r dr}{c}\cos\alpha = \frac{2P}{c\delta^2}\int\left(1-\frac{\alpha^2}{2}\right)rdr$

$= \frac{2P}{c\delta^2}\int_0^{\delta}\left(1-\frac{(n-1)r^2}{2R^2}\right)rdr$, Because $\theta = \frac{r}{R} = \frac{\alpha}{n-1}$

$= \frac{P}{c}\left[1-\frac{(n-1)\delta^2}{4R^2}\right]$

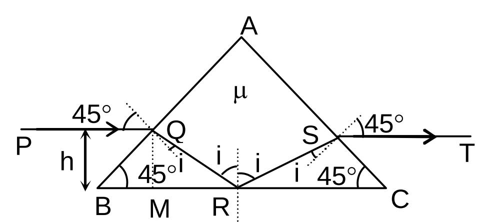
So, $P = \frac{mg4R^2c}{(n-1)^2\delta^2}$

No. of photons/time = $\frac{P}{hf}$

SECTION – C

13. 1

Sol. $\sin 45° = \mu \sin r \Rightarrow r = 30°$



From ΔBQR and ΔCSR

$\frac{BQ}{SC} = \frac{BR}{RC}$

$\therefore\ h' = 3\left(\frac{RC}{BR}\right)$

i = 45°+ r = 75°

So, $\frac{QM}{MR} = \tan 15°$, so MR = 12 cm

So, BR = 15 cm and RC = 5 cm

So, h′ = 1cm

14. 9

Sol. In Ist case: $\frac{1}{v_1} - \frac{1}{-3} = \frac{1}{2} \Rightarrow v_1 = 6$ cm

When one lens is removed, the new focal length of the objective is

$\frac{1}{f'} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10} \Rightarrow f' = 2.5$ cm

The new position of image is

$\frac{1}{v_2} - \frac{1}{-3} = \frac{1}{2.5}$

$\therefore\ v_2$ = 15 cm

So, shifting = 15 – 6 = 9 cm

15. 2

Sol. $m_A c^2 + K_A + m_B c^2 + K_B = m_C c^2 + K_C + \text{excitation energy}$

So, $K_C = 2\,\text{MeV}$

16. 9

Sol. Intensity at the surface of sphere = $2\pi^2 f^2 (\Delta R)^2 \rho \sqrt{\frac{B}{\rho}}$

So, total power = $2\pi^2 f^2 (\Delta R)^2 \rho \sqrt{\frac{B}{\rho}} 4\pi R^2$

= $8\pi^3 \sqrt{\rho B} f^2 R^2 (\Delta R)^2 = 9$ Watt.

17. 6

Sol. $P_2 - P_1 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) = \frac{1}{2}\rho\left(\frac{A_2^2 V_2^2}{A_1^2} - V_2^2\right)$

$= \frac{1}{2}\rho V_2^2 A_2^2 \left(A_1\right)^{-2} - \frac{1}{2}\rho V_2^2$

From the graph,

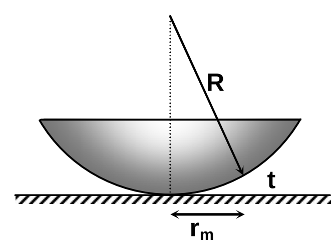
$\frac{1}{2}\rho V_2^2 A_2^2 = \frac{300\times10^3}{16}$ N/m^2 and $\frac{1}{2}\rho V_2^2 = 300\times10^3$ N/m^2

So, $A_2 = \frac{1}{4}\text{m}^2$ and $V_2 = \sqrt{600}$ m/s

So, volume rate Q = A_2V_2 = 6 m^3/s

18. 4

Sol. For newton's ring, $t = \frac{r_m^2}{2R}$



For fringe system to disappear completely

$2t = m\lambda_1 = \left(m + \frac{1}{2}\right)\lambda_2$

So, $\frac{2t}{\lambda_1} - \frac{2t}{\lambda_2} = \frac{1}{2}$

So, $2t\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2} = \frac{1}{2}$

So, $2\frac{r_m^2}{2R}\frac{2\times10^7}{16\times10^{-14}} = \frac{1}{2}$

$\therefore r_m = 4 \times 10^{-2}$ m

19. 2

Sol. Refraction at the air and core (at x = 0) interface gives

(1) sin θ_i = n_1 sin r

Now for refraction at various parallel interfaces at different x.

$n_1 \cos r = n_1 \sin\left(\frac{\pi}{2} - \theta\right) = n \cos\theta$

So, $n \cos\theta = n_1\sqrt{1 - \frac{\sin^2\theta}{n_1^2}}$

$= \sqrt{n_1^2 - \sin^2\theta_i} = = \sqrt{\frac{9}{4} - \frac{1}{4}} = \sqrt{2}$

20. 3

Sol. Both the lenses must be converging

If x is the distance of object from the first lens

So, $m_1 = \frac{f_1}{f_1 - x}$ where f_1 is focal length of first lens

and $m_2 = \frac{f_2}{f_2 - \left(L - \frac{xf_1}{f_1 - x}\right)}$ where f_2 is focal length of the second lens and L is separation between the lenses.

So, $\frac{1}{M_1} = \frac{1}{m_1 m_2} = \frac{f_1 - x}{f_1} \frac{f_2 - \left(L - \frac{xf_1}{x - f_1}\right)}{f_2} = 1 - \frac{L}{f_2} - \frac{x(f_1 + f_2 - L)}{f_1 f_2}$

When the lenses are interchanged, in the above expression only the second term changes

So, $\frac{1}{M_2} = 1 - \frac{L}{f_1} - \frac{x(f_1 + f_2 - L)}{f_1 f_2}$

$\therefore \frac{1}{M_1} - \frac{1}{M_2} = L\left(\frac{1}{f_1} - \frac{1}{f_2}\right) = L(D_1 - D_2)$

So, $1 - \frac{1}{4} = \frac{1}{4}(D_1 - D_2)$

$\therefore D_1 - D_2 = 3$ diopter

Chemistry

PART – II

SECTION – A

21. A, B, C, D
Sol. Facts.

22. A, B, C, D
Sol. Facts.

23. B
Sol. $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$
100 gm sample contains 80 gm $CaCO_3$
$\therefore$ Moles of $CaCO_3 = \frac{80}{100} = 0.8$
$\therefore$ Moles of CaO formed = 0.8 mole
Molarity $= \frac{0.8}{200} \times 1000 = 4.0$

24. A, B, D

25. A, B, C, D

26. A, B, C
Sol. For feasibility of reaction $E^o = +ve$ & $\Delta G^o = -ve$

$[Fe \longrightarrow Fe^{2+} + 2e^- \quad] \times 5$

$[MnO_4^- + 5e^- + 8H^+ \longrightarrow Mn^{2+} + 4H_2O] \times 2$

$5Fe + 2MnO_4^- + 8H^+ \longrightarrow 2Mn^{2+} + 5Fe^{2+} + 8H_2O$

$\Delta G_1^o = -2 \times F \times 0.44$

$\Delta G_2^o = -5 \times F \times 1.51$

$\Delta G_3^o = -10 \times FE^o = 5\Delta G_1^o + 2\Delta G_2^o$

$E^o = 0.44 + 1.51 = 1.95$

$Fe \longrightarrow Fe^{3+} + 3e^-$

$MnO_4^- + 5e^- + 8H^+ \longrightarrow Mn^{2+} + 8H_2O$

$E^o = 0.036 + 1.51 = 1.546$

$Fe^{2+} \longrightarrow Fe^{3+} + e^-$

$MnO_4^- + 5e^- + 8H^+ \longrightarrow Mn^{2+} + 8H_2O$

$5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow Mn^{2+} + 8H_2O + Fe^{3+}$

$E^o = (-0.771) + (1.51) = 0.739$

$Fe^{2+} \longrightarrow Fe^{3+} + e^- \quad E^o = -0.771$

$Sn^{4+} + 2e^- \longrightarrow Sn^{2+} \quad E^o = +0.15$

$2Fe^{2+} + Sn^{4+} \longrightarrow 2Fe^{3+} + Sn^{2+}$

$E^o = (+0.15) + (-0.771) = -0.621$

27. A

Sol. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$8/6 = \gamma$

$$T_2 = \frac{T_1(1)^{2/6}}{(27)^{2/6}} = \frac{T_1}{3} = \frac{300}{3} = 100$$

28. A, C

Sol. $\Delta G = \Delta H - T\Delta S$

$$T\Delta S = 2.303nRT\log\frac{V_2}{V_1}$$

$$= 2.303 \times 1 \times 8.314 \times 300\log\frac{10}{1}$$

$= 5744.14\ J\ mol^{-1}\ K^{-1}$

29. B

30. A, B, C, D

Sol. **(for Q. 29 to 30)**

$$Mn(NO_3)_2 \xrightarrow{\Delta} MnO_2 + 2NO_2$$

$$MnO_2 + Conc.\ H_2SO_4 \longrightarrow MnSO_4 + O_2 + H_2O$$

$$MnSO_4 + NaOH \longrightarrow \underset{(White)}{Mn(OH)_2} \xrightarrow{O_2} \underset{(Brown)}{MnO(OH)_2}$$

$$PbO_2 + MnSO_4 \xrightarrow{H^+} \underset{(Purple)}{HMnO_4} + Pb^{2+}$$

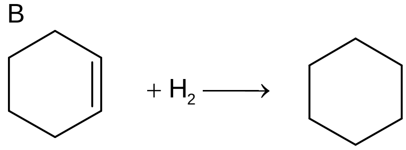
$$Mn^{2+} + NaBiO_3 \xrightarrow{H^+} HMnO_4 + Bi^{3+}$$

$$Mn^{2+} + IO_4^- \xrightarrow{H^+} HMnO_4 + IO_3^-$$

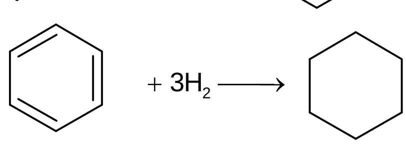
$$NO_2 + KI \xrightarrow[Paper]{Starch} Blue\ spot$$

31. B

Sol.

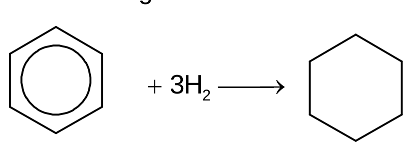


$\Delta H = -28.6\ kcal/mole$



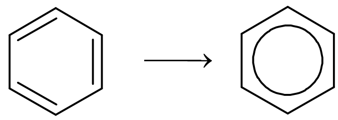
$\Delta H = (-28.6) \times 3\ kcal/mole$

Non resonating



$\Delta H = -49.8\ kcal/mole$

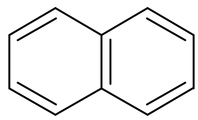
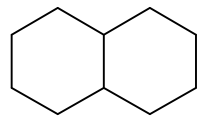
Benzene



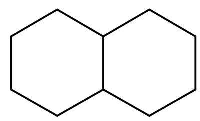
$$\Delta_{resonance}H = (-85.8) - (-49.8) = -36 \text{ kcal/mole}$$

32. A, B

Sol.

 $+ 5H_2 \longrightarrow$  $\Delta H = 5 \times (-28.6) = -143 \text{ kcal/mole}$

Stable resonating structure

Naphthalene $+ 5H_2 \longrightarrow$  $\Delta H = -82 \text{ kcal/mole}$

$\therefore \Delta_{resonance} H_{naphthalene} = (-143) - (-82) = -61$

Resonance energy of benzene = - 36

One naphthalene ring equivalent to two benzene ring

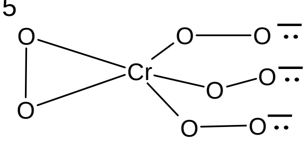
∴ Magnitude of resonance energy of naphthalene (-61), is less than two benzene ring (-72).

So benzene ring is more stable.

SECTION – C

33. 5

Sol.



34. 3

Sol. $\frac{r^+}{r^-} = 0.155 - 0.225$, trigonal planar.

35. 7

Sol. $W = zit = \frac{63.5}{2 \times 96500} \times 6 \times 59 \times 60$

= 7 gm

36. 5

Sol. $N = \frac{28}{5.6} = 5$

37. 8

Sol. $\%SO_3 = \frac{101.8 - 100}{18} \times 80 = 8$

38. 5

Sol. $x = \frac{5}{3}$

$\therefore y = \frac{5}{3} \times 5 = 5$

39. 7

Sol. $760 = p_A^o X_A + p_B^o X_B$

$760 = 500X_A + 900(1 - X_A)$

$X_A = 0.35$

$\therefore n = 0.35 \times 20 = 7$

40. 4

Sol. $2Al + \frac{3}{2}O_2 \longrightarrow Al_2O_3 \quad \Delta H = -399.0 \text{ kcal}$

$2Fe + \frac{3}{2}O_2 \longrightarrow Fe_2O_3 \quad \Delta H = -199.0 \text{ kcal}$

By using these. We get

$2Al + Fe_2O_3 \longrightarrow Al_2O_3 + 2Fe \quad \Delta H = -200 \text{ kcal}$

Vol. of fusion mixture = vol. of Al + vol. of Fe_2O_3 $= \frac{54}{3} + \frac{160}{5} = 50 \text{ ml}$

$\therefore$ Fuel value $= \frac{200}{50} = 4 \text{ kcal/ml}$

Mathematics

PART – III

SECTION – A

41. A, B, C, D

Sol. Let $f(x) = x^n - 1 = \prod_{\alpha \in U_n} (x - \alpha)$

$$A = \prod_{\alpha \in U_n}\left(\alpha + \frac{1}{\alpha}\right) = \prod_{\alpha \in U_n}\frac{\alpha^2+1}{\alpha} = \frac{\prod_{\alpha \in U_n}(\alpha+i)(\alpha-i)}{\prod_{\alpha \in U_n}\alpha} = \frac{\left((-i)^n-1\right)\left((i)^n-1\right)}{(-1)^{n-1}}$$

42. A, D

Sol. $$R_n = \begin{vmatrix} 3 & 1 & 0\,0 \ldots \\ 1 & 3 & 1\,0\,0 \ldots \\ 0 & 1 & \\ . & 0 & \\ . & . & \end{vmatrix}_{n\times n} \Rightarrow R_n = 3\begin{vmatrix} 3 & 1 & 0 \ldots \\ 1 & & \\ 0 & & \\ . & & \end{vmatrix}_{(n-1)\times(n-1)} + \begin{vmatrix} 1 & 1 & 00 \ldots \\ 0 & 3 & \\ 0 & 1 & \\ . & 0 & \\ . & . & \end{vmatrix}_{(n-1)\times(n-1)}$$

$\Rightarrow R_n = 3R_{n-1} - R_{n-2}$

$\Rightarrow R_n + R_{n-2} = 3R_{n-1}$

43. B, C, D

Sol. $P^T(P+Q)Q^T = (P+Q)^T \Rightarrow |P + Q| = 0,\ P^T(P-I) = -(P-I)^T$

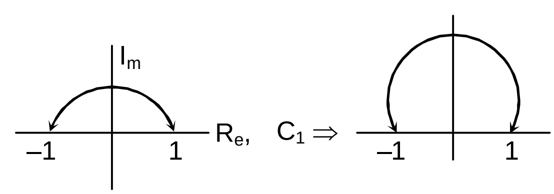
Either $|P|\,|P - I| = -|P - I|$ or $|Q||Q - I| = -|Q - I|$, because $|P|\,|Q| < 0$

$P\left(P^{-1}+Q^{-1}\right)Q = P+Q \Rightarrow \left|P^{-1}+Q^{-1}\right| = -\left|P+Q\right| = 0$

44. B, C

Sol. Shown in the figure

$C_2 \Rightarrow$ … R_e, $C_1 \Rightarrow$ … , $|\beta| \neq 1$



45. B, C, D

Sol.

Red box	Green box
5 balls	9 balls
R_g green	$8 - R_g$ green
$5 - R_g$ red	$R_g + 1$ red

So, $G_r = R_g + 1$

$R_g + G_r = 2R_g + 1$ (cannot be even number)

$1 \le 2R_g + 1 \le 11$

For $R_g + G_r$ non-prime $\Rightarrow R_g = 0$ or $4 \Rightarrow$ so the probability = $\dfrac{{}^6C_5 + {}^8C_4 \times {}^6C_1}{{}^{14}C_5} = \dfrac{213}{1001}$

46. A, B, C, D

Sol. The each number $a_1a_2 \ldots\ldots a_5$ will correspond to a term in expansion

$\left(a_1^1 + a_1^2 + \ldots\ldots + a_1^7\right)\left(a_2^1 + a_2^2 + \ldots\ldots + a_2^7\right)\left(a_3^1 + \ldots\ldots + a_3^7\right)\ldots\ldots\left(a_5^1 + a_5^2 + \ldots\ldots + a_5^7\right)$

So find sum of coefficient of x^{3k}, x^{2k}, x^{4k} in expansion of $\Rightarrow \left(x^1 + ... + x^7\right)^5$, for the A, B, C respectively (where $k \in N$), if k = 33 then number possible case will be $5 + {}^5C_2 = 15$

47. B, D

Sol. $P = \frac{{}^{\omega}C_3}{{}^{\omega+r}C_3}$, $\frac{4}{3}P = \frac{{}^{\omega+1}C_3}{{}^{\omega+r+1}C_3}$

Equality value of P, $r = \frac{-\omega^2 + \omega + 2}{\omega - 11}$

$r = -\omega - 10 + \frac{108}{11 - \omega}$

$r_{max} = 88$, ω can not 6 as r will not be integer

48. B, C, D

Sol. There are 8 steps between (0, 0) and (3, 5) so A and B can be meet at (0, 4), (1, 3), (2, 2) and (3, 1) point

Let a_i and b_i denote the number of paths to P_i from (0, 0) to (3, 5) respectively, because A has to take r steps to right and B has to take r + 1 steps down the number of ways A and B can meet at P_i is

$a_i \cdot b_i = {}^4C_i \cdot {}^4C_{i+1}$, because A and B can each take 2^4 path in 4 ways, so probability they will meet is $\frac{1}{2^8}\sum_{i=0}^{3} a_i b_i$

49. A, C

50. C, D

Sol. $xyz = \pm(2^3 \times 3^2 \times 5)$, number of possible triplet of (x, y, z) are $= {}^5C_2 \times {}^4C_2 \times {}^3C_2 = 180$

so $2\alpha = 8 \times 180 = 1440$, if x + y + z = even so possible cases are odd + odd + even, even + even + even

For odd + odd + even, total number of cases = 54

For even + even + even, total number of cases = 18, so $\beta = 72$

Trace of $A^3 = x^3 + y^3 + z^3$, $x^3 + y^3 + z^3 = 3xyz \Rightarrow$ Either x = y = z or x + y + z = 0 , so $\gamma = 0$

51. A, C

Sol. f(1) + f(2) + f(3) + f(4) = 2000, By A.M. $\geq$ G.M., $A^4 \geq 6144$ f(1) f(2) f(3) f(4)

Only equality holds true so f(1) = 2f(2) = 3f(3) = 4f(4)

so f(1) = 960, f(2) = 480, f(3) = 320, f(4) = 240

52. B, C, D

Sol. Let $g(x) = (x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$, $\alpha, \beta, \gamma, \delta$ are the roots of polynomial g(x) = 0

$1 = \sqrt{1+\alpha^2}\sqrt{1+\beta^2}\sqrt{1+y^2}\sqrt{1+\delta^2} \Rightarrow \alpha = \beta = \gamma = \delta = 0$

So, $g(x) = x^4$ and f(1) = d, f(2) = c, f(3) = b, f(4) = a

SECTION – C

53. 2

Sol. A = B + I, $B = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}$, $B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

54. 4

Sol. $f'(x) = 0$, has five distinct and real roots

55. 0

Sol. $\sum a = 2$, $\sum ab = -1$, abc = –2

So, a, b, c are roots and equation $x^3 - 2x^2 - x + 2$

$\Rightarrow$ (a, b, c) = (1, –1, 2)

56. 7

Sol. $\frac{1}{R_n} = \frac{1}{4}\left(\sqrt{n^2 + (n+1)^2} - \sqrt{n^2 + (n-1)^2}\right)$, find R_1, R_2, ..., R_{20} it will be a telescopic sum

57. 2

Sol. Form a cube of edge length 3, $R = \frac{(\text{length of body diagonal})}{2} = \frac{3\sqrt{3}}{2}$

58. 8

Sol. Point Q(8, 3, –7) lies on the line $\frac{x-1}{7} = \frac{y-2}{1} = \frac{z+2}{-5}$, let P is point of intersection of the given lines P(–6, 1, 3)

S is sphere with radius PQ and S′ is sphere with diameter PQ

59. 0

Sol. If possible value the value of det P is k, then –k al also possible for each positive k

60. 2

Sol. Let a_n is total of such subset for a n number of element set . So in that subset let either 1, is appeared or not, if 1 appeared then total number of subset will be a_{n-2}, if 1 is not appeared total number of subset is a_{n-1},

so $a_n = a_{n-1} + a_{n-2}$ provide $a_1 = 1$, $a_2 = 2$ find $a_9 = 55$