FIITJEE

ALL INDIA TEST SERIES

OPEN TEST

JEE (Advanced)-2019

PAPER – 2

TEST DATE: 03-02-2019

ANSWERS, HINTS & SOLUTIONS

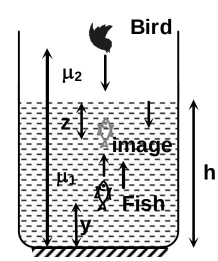
Physics

PART – I

SECTION – A

1. B, D

Sol. Distance of image of fish as seen by bird only after refraction is



$$Z = (x - h) + (h - y)\frac{\mu_2}{\mu_1}$$

$$Z = (x - h) + (h - y)\frac{1}{4/3}$$

Differentiate w.r.t. time

$$\frac{dZ}{dt} = (-6 + 2) + (-2 - 4)\frac{3}{4} = -\frac{17}{2} \text{ cm/sec}$$

Similarly we can calculate for 2^{nd} case.

2. C, D

Sol. Collision takes place in ideal gases are elastic hence total kinetic energy will be conserved.

3. A, C, D

Sol. $T = \frac{2u\sin\theta}{g} = 2\sin 15°$

Horizontal component of velocity will reverse in each collision.

$$R = \frac{u^2 \sin 2\theta}{g} = 5 \text{ m}$$

4. A, C

Sol. In time interval (0 to 3) sec, $S = ut + \frac{1}{2}at^2$ = 30 m

$\vec{v}_{average} = \frac{30}{3} = 10$ m/s

In time interval (0 to 6) sec

$v = u + at = 40$ m/s

$\left|\vec{a}_{average}\right| = \frac{\left|\vec{v}_f - \vec{v}_i\right|}{t} = \frac{(5\pi + 40) + 5\pi}{6} = \frac{5\pi + 20}{3}$ m/s^2

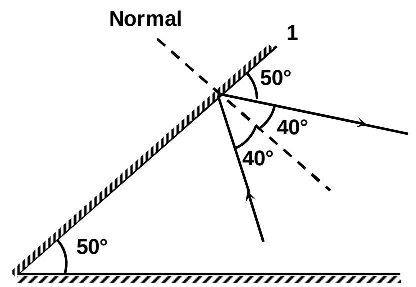
5. A, C

Sol. Deviation

$\delta_1 = \pi - 2\theta$ (Clockwise)

$\delta_2 = \pi - 2(50 + \theta)$ (Anti-clockwise)

$\delta = \delta_1 - \delta_2$



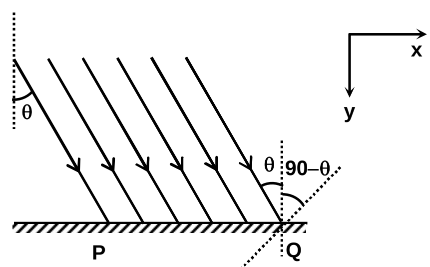
6. A

Sol. Ball moves very slowly hence it is in equilibrium at each instant.

7. D

Sol. Intensity is given by $I = \frac{N\left(\frac{hC}{\lambda}\right)}{(A\cos\theta)\Delta t}$, where

N → total number of photons and λ → wavelength of photons



(A) $F_x = \frac{\Delta P_x}{\Delta t} = \frac{IA\cos\theta \cdot \sin\theta}{C}$

$F_y = \frac{\Delta P_y}{\Delta t} = \frac{IA\cos\theta\cos\theta}{C}$

$p_r = \frac{F_y}{A} = \frac{I\cos^2\theta}{C}$

(B) $F_x = 0$

$F_y = \frac{2IA\cos^2\theta}{C}$

$p_r = \frac{2I\cos^2\theta}{C}$

(C) and (D)

$F_x = \frac{IA\cos\theta\sin\theta}{C}(1 - r)$

$F_y = \frac{IA\cos^2\theta}{C}(1 + r)$

$p_r = \frac{I\cos^2\theta}{C}(1 + r)$

8. C

Sol. Circuit can be solved by using Phasor diagram.

9. A

Sol. By using concept of motional emf induced in the circuit.

10. A

Sol. $\Rightarrow$ $v_0 = \sqrt{2gh} = 5$ m/s …(i)

$\Rightarrow$ $J_y = \Delta P_y = 2 \times 80 \times 5 = 800$ N-s …(ii)

$\Rightarrow$ $J_x = \Delta P_x$

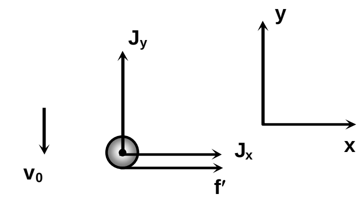
$\mu J_y = mv_x$

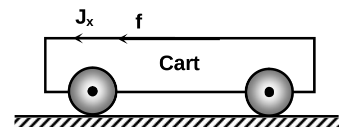
$v_x = 1$ m/s …(iii)

$\Rightarrow$ $Mv_C = J_x$

$v_C = 0.4$ m/s …(iv)

$\Rightarrow$ $t_0 = \frac{2 \times 5}{10} = 1\,\text{sec}$ …(v)





$\Rightarrow$ $L_{min} = 2(1 + 0.4)\ 1 = 2.8$ m …(vi)

$\Rightarrow$ At the time of second collision

$R\omega' - 1 = 0.4$

$\omega' = 7$ rad/s …(vii)

$\Rightarrow$ $\vec{J}' = \Delta\vec{L}$

$-J_x R = I(\omega' - \omega_0)$

$\omega_0 = 19.5$ rad/sec …(viii)

$\Rightarrow$ $W_{m\to M} = \frac{1}{2}Mv_C^2 = 16\text{J}$ …(ix)

$\Rightarrow$ $W_{M\to m} = \frac{1}{2}I(\omega')^2 + \frac{1}{2}mv_x^2 - \frac{1}{2}I\omega_0^2 = -172$ J

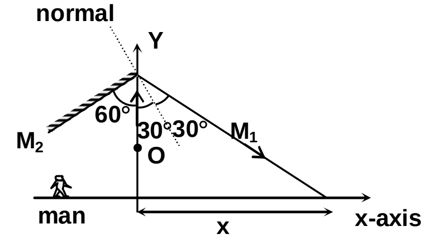
SECTION – D

11. 00006.92

Sol. $\tan 60° = \frac{x}{2}$

$x = 2\sqrt{3}$ m

Hence total length = $4\sqrt{3}$ m.



12. 00035.20

Sol. To reach at ‘B’ it should crosses the point ‘A’. Now from work energy theorem

$W = \Delta K$

$W_g + W_f = K_f - K_i$

$-mg(4H) - (0.4)mgH = 0 - k_i$

$K_i = 44$ mH = 35.20

13. 00002.40

Sol. $\vec{L} = I_{cm}\vec{\omega} + (\vec{r} \times \vec{p}_{cm})$

$|\vec{L}| = \frac{m(2R)^2}{2}\omega_0 + R(2m)(R\omega_0)$

$|\vec{L}| = 4mR^2\omega_0 = 2.40$

14. 00016.64

Sol. $d\sin\theta \pm (\mu-1)t = \Delta x$

$d\left(\frac{3\lambda}{2d}\right) \pm \frac{3\lambda}{2} = \Delta x$

$\Delta x = 3\lambda$

Hence maximum intensity will occur at 'O'.

$I = 4I_0 = 16.64\ W/m^2$

15. 00002.67

Sol. $(Mg) \times 1 = \frac{2E_0}{tc} \times 2$

$M = \frac{2 \times 20 \times 2}{10 \times 0.01 \times 10^{-3} \times 3 \times 10^8} = \frac{8}{3} \times 10^{-3} kg$

16. 00000.74

Sol. pitch = 0.2 mm

Total division = 200

Least count = 0.001 mm

–ve zero error = 40 × L.C. = 0.04 mm

Reading = 0.6 mm + 100 × L.C. = 0.7 mm

Thickness = 0.7 mm + 0.04 mm = 0.74 mm

17. 00002.25

Sol. In time dt shift in centre of mass of system (ball + liquid)

$dS_{CM} = \frac{m_1 ds_1 + m_2 ds_2}{M} = \frac{(\rho_b V)vdt - (\rho_\ell V)vdt}{M}$ …(i)

Momentum of system = $M.\frac{dS_{cm}}{dt}$ …(ii)

$= (\rho_b V)\,v - (V\,\rho_\ell)\,v$

Therefore momentum of liquid = $(V\,\rho_\ell)v$

$= 2.25\ gm/cm^3$.

18. 00002.33

Sol. Speed of projection $(v_0) = \sqrt{\frac{2GM}{R}}$ ...(1)

$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}$ …(2)

$v = \sqrt{\frac{2GM}{r}}$

$\int_R^{4R} \sqrt{r}\,dr = \sqrt{2GM}\int_0^t dt$

$t = \frac{7}{3}\sqrt{\frac{2R^3}{GM}}$.

Chemistry

PART – II

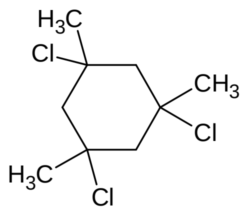
SECTION – A

19. A, C

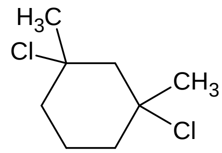
Sol. Main hexagonal unit cell (figure A) consists of three identical smaller unit cells (B).

20. A, C, D

Sol.



(A)



(B)

21. B, C, D

22. B, C

Sol. Weight of pay load = 80×10^3 g

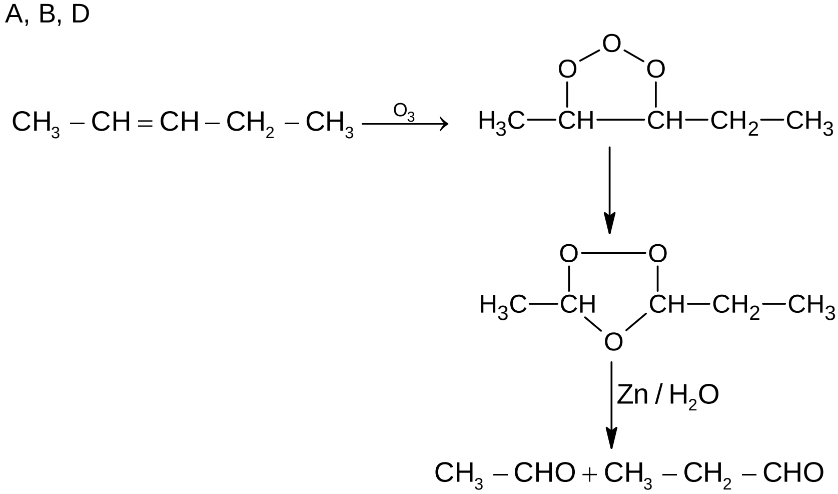
Let, the number of balloons required be x.

$$80 \times 10^3 + 0 + 100x = \frac{nRT}{P} x \times 1.25$$

x = 26.8

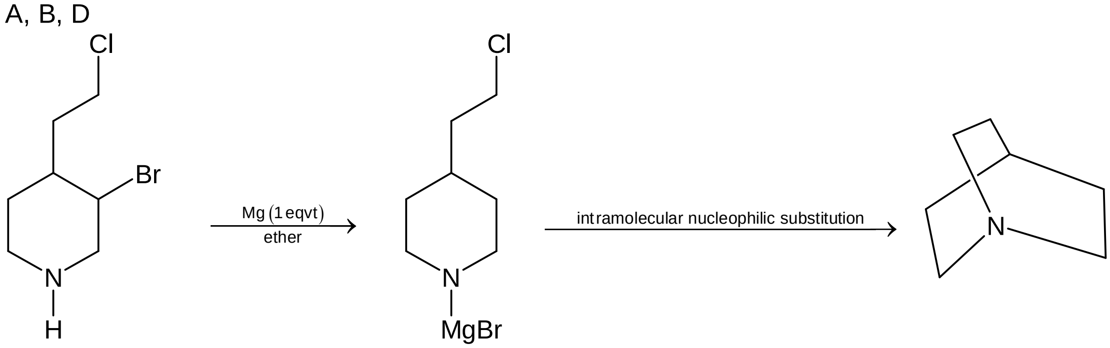
23. A, B, D

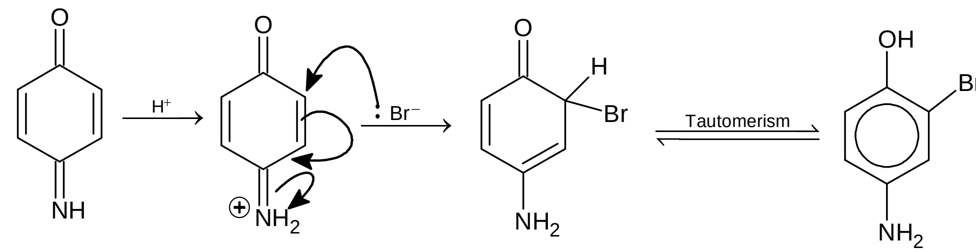
Sol.



24. A, B, D

Sol.





25. A
26. C

Sol.

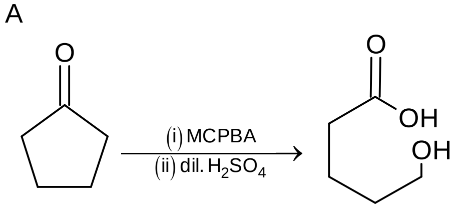
	NaCl type solid	CsCl type solid
1st nearest distance	$\frac{a}{2}$	$\frac{\sqrt{3}a}{2}$
2nd nearest distance	$\frac{a}{\sqrt{2}}$	a
3rd nearest distance	$\frac{\sqrt{3}a}{2}$	$\sqrt{2}a$
4th nearest distance	a	$\frac{\sqrt{11}a}{2}$
5th nearest distance	$\frac{\sqrt{5}a}{2}$	$\sqrt{3}a$
6th nearest distance	$\frac{\sqrt{3}a}{\sqrt{2}}$	2a

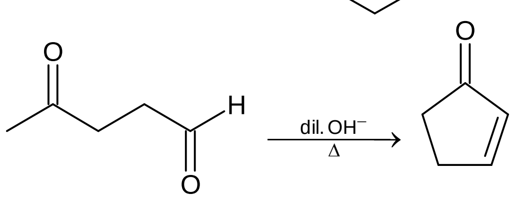
27. C

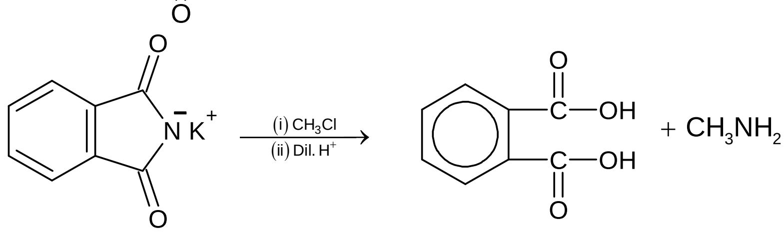
Sol. General property trends.

28. A

Sol.







$$CH_3NC \xrightarrow{Na/C_2H_5OH} CH_3 - NH - CH_3$$

SECTION – D

29. 00002.50

Sol. $\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$

n = 1

$\left(P+\frac{a}{V^2}\right)(V-b)=RT$

If 'b' is negligible, then

$\left(P+\frac{a}{V^2}\right)V=RT$, $P=\frac{RT}{V}-\frac{a}{V^2}$

The equation is quadratic in 'V', thus

$V=\frac{+RT\pm\sqrt{R^2T^2-4aP}}{2P}$

'V' has one value at a given P and T, thus numerical value of discriminant = 0

So, $R^2T^2=4aP$

$P=\frac{R^2T^2}{4a}=\frac{(0.08)^2\times(300)^2}{4\times3.6}=40$

30. 00000.50

Sol. $\psi_{(x)}=A\sin\left(\frac{2\pi x}{\lambda}\right)$

$\frac{d\psi_{(x)}}{dx}=\frac{2\pi}{\lambda}A\cos\left(\frac{2\pi x}{\lambda}\right)$

$\frac{d^2\psi_{(x)}}{dx^2}=-\left(\frac{2\pi}{\lambda}\right)^2A\sin\left(\frac{2\pi x}{\lambda}\right)$ So, n = 2, m = -4, p = 2, q = 2

$\frac{d^2\psi_{(x)}}{dx^2}=-\frac{4\pi^2\psi_{(x)}}{\lambda^2}$

31. 00015.05

Sol.

Complex	Magnetic moment
$\left[Cu(NH_3)_4\right]^{2+}$	$\sqrt{3}=1.73$ B.M.
$\left[Fe(CN)_6\right]^{3-}$	$\sqrt{3}=1.73$ B.M.
$[NiCl_4]^{2-}$	$2\sqrt{2}=2.82$ B.M.
$[CoF_6]^{3-}$	$\sqrt{24}=4.90$ B.M.
$\left[Fe(H_2O)_5(NO)\right]SO_4$	$\sqrt{15}=3.87$ B.M.

32. 00054.50

Sol. x = 22.5; y = 30; z = 2

In parallel reactions the activation energy of the overall path is given by

$(Ea)_{net}=\frac{k_1Ea_1+k_2Ea_2}{k_1+k_2}$

33. 00007.50

Sol. $2\left[Au\left(CN\right)_2\right]^- + Zn \longrightarrow 2Au + \left[Zn\left(CN\right)_4\right]^{2-}$

$E^o = -0.60 - \left(-1.26\right) = 0.66\ V$

Since E^o value for the reduction of complexed Ag^+ ion to Ag is higher than that of complexed Au^+ ion into Au. So, Ag^+ ion will be reduced first.

Moles of Zn added $= \frac{78}{65} = 1.2$

Moles of $[Ag(CN)_2]^-$ = 0.003 × 500 = 1.5
Moles of $[Au(CN)_2]^-$ = 0.002 × 500 = 1.0
Moles of Zn used to reduce 1.5 moles of $[Ag(CN)_2]^-$ = 0.75
Moles of $[Au(CN)_2]^-$ reduced by remaining 0.45 moles of Zn = 0.90
Moles of $[Au(CN)_2]^-$ left in the solution = 0.10

Concentration of $[Au(CN)_2]^-$ left in the solution $= \frac{0.10}{500} = 2 \times 10^{-4}\ M = x$

Also, $Au^+ + 2CN^- \rightleftharpoons \left[Au\left(CN\right)_2\right]^-$, $K_f = 4 \times 10^{28}$

$$K_f = \frac{\left[Au\left(CN\right)_2\right]^-}{\left[Au^+\right]\left[CN^-\right]^2} = \frac{90\left[Au^+\right]}{10\left[Au^+\right]\left[CN^-\right]^2} = 4 \times 10^{28}$$

$\left[CN^-\right] = 1.5 \times 10^{-14}\ M = y$

So, $\frac{y}{x} = \frac{1.5 \times 10^{-14}}{2 \times 10^{-4}} = 7.50 \times 10^{-11}$

So, A = 7. 50

34. 00367.50

Sol. Let, the moles of $KClO_3$ required to be 'x'. Assuming 50% yield in each step.

Moles of O_2 formed in step (i) $= \frac{x}{2}$.

Moles of each of K_2MnO_4 and Cl_2 formed in (i) $= \frac{x}{4}$

Moles of O_2 formed in step (ii) = $\frac{x}{8}$

Total moles of O_2 formed $\frac{x}{2} + \frac{x}{8} = \frac{5x}{8}$

$\frac{5x}{8} = \frac{60}{32}$

x = 3

35. 00000.80

Sol. $\Delta T_f = i \times K_f \times m$

0.00558 = i × 1.86 × 0.001 $\quad \therefore i = 3$

$i = 1 + \left(y - 1\right) \times 1 = 3 \quad \therefore y = 3$

So, formula of the complex $\left[Pt\left(NH_3\right)_4 Cl_2\right]Cl_2$

36. 00004.50

Sol. x = 3, y = 15, z = 10

Mathematics PART – III

SECTION – A

37. A, B, C, D

Sol. $A + B = ABAB = A^2 + A$

$\Rightarrow B = A^2$

$BAB = A^5 = A + I \Rightarrow A(A^4 - I) = I$

$B^5 - A^5 = (A^5)^2 - A^5 = (A + I)^2 - (A + I) = A^2 + A = A + B$

38. A, D

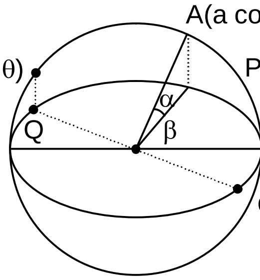
Sol. Let f(a) = 0 and $F(x) = \int_0^x |f(t)|dt$

$F'(x) = -f(x)\ 0 \le x < a,\ F'(x) = f(x)\ a \le x \le 1$

$$\Rightarrow \int_0^1 f(x)\left(\int_0^x |f(t)|dt\right)dx = \frac{(F(1))^2}{2} - (F(a))^2 = \frac{(S_1 + S_2)^2}{2} - (S_2)^2 = 7$$

39. B, C

Sol. $$\tan\alpha = \frac{\tan\theta - \frac{b}{a}\tan\theta}{1 + \frac{b}{a}\tan^2\theta} = \frac{1 - \frac{b}{a}}{\cot\theta + \frac{b}{a}\tan\theta} \le \frac{1 - \frac{b}{a}}{2\sqrt{\frac{b}{a}}} = \frac{a-b}{2\sqrt{ab}}$$



$$\tan\beta = \frac{\frac{b}{a}\tan\theta + \frac{b}{a}\cot\theta}{1 - \frac{b^2}{a^2}} \ge \frac{\frac{2b}{a}}{1 - \frac{b^2}{a^2}} = \left(\frac{2ab}{a^2 - b^2}\right)$$

40. A, C

Sol. $$n^2 - 5(2n - 5) \le \frac{393(n-5)}{5} \le n^2 - 25$$

$\Rightarrow 73.6 \le n \le 83.6$, n = 80 is only possible value

41. B, C

Sol. $$\lim_{x\to 0} \frac{\sqrt{1+x^2}\tan\left(\frac{x}{\sqrt{1+x^2}}\right) + 2\sqrt{1-x^2}\sin\frac{x}{\sqrt{1-x^2}} - 3x}{x^p}$$

Using expansion

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}\left(\frac{x}{\sqrt{1+x^2}}+\frac{x^3}{3\left(1+x^2\right)^{\frac{3}{2}}}+\frac{2}{15}\frac{x^5}{\left(1+x^2\right)^{\frac{5}{2}}}.....\right)+2\sqrt{1-x^2}\left(\frac{x}{\sqrt{1-x^2}}-\frac{x^3}{3!\left(1-x^2\right)^{\frac{3}{2}}}+\frac{x^5}{5!\left(1-x^2\right)^{\frac{5}{2}}}.....\right)-3x}{x^p}$$

$$\lim_{x\to 0}\frac{\left(-\frac{2}{3}+\frac{2}{15}\frac{1}{\left(1+x^2\right)^2}+\frac{2}{5!\left(1-x^2\right)^2}\right)x^5}{x^p}$$

$\Rightarrow$ p = 5 and limit = $-\frac{2}{3}+\frac{2}{15}+\frac{2}{5!}=\frac{-31}{60}$

42. A, B, D

Sol. All the values of |A| = ±4, 0 and determinant is 0 when two rows/columns are proportional

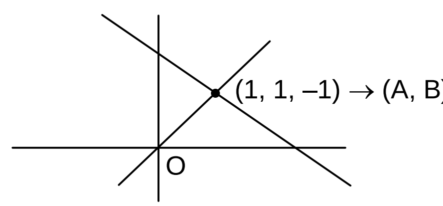
43. B

Sol. (P) $d = 2\sqrt{3}$

(Q) These lines are skew and O lies on shortest distance

(R) Lines are parallel and O lies mid way between them

(S) Lines are coplanar and perpendicular



44. D

Sol. We can consider 9 cells as 9 different boxes and we have to fill these boxes by 3 identical balls (2 written on them), 4 identical balls (3 written on them) and 7 identical balls (5 written on them) as per given conditions

45. A

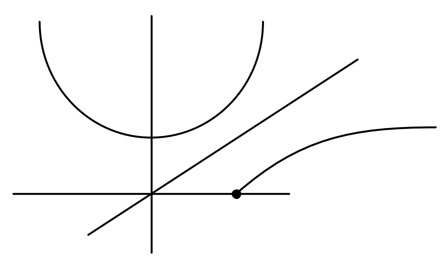
Sol. (P) $x^2 - a = x$

$x^2 - x - a = 0$

$D < 0$

$1 + 4a < 0$

$a < -\frac{1}{4}$



(Q) $-\sqrt{a} < -a$

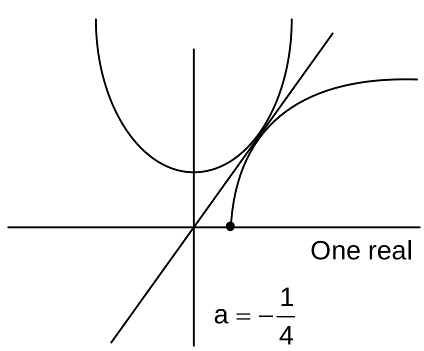
$\sqrt{a} > a$

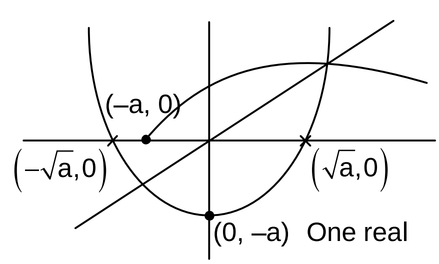
$a > a^2$

$a^2 - a < 0$

$a(a - 1) < 0$

$0 < a < 1$





(R) $x^2 - x - a = 0$

$1 + 4a > 0, -a > 0$

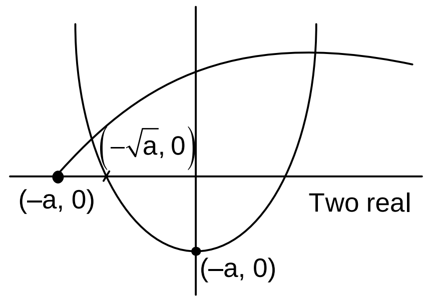
$a > -\frac{1}{4}, a < 0$

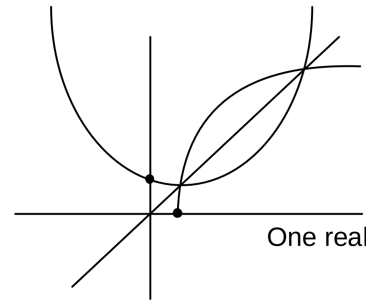
$-\frac{1}{4} < a \le 0$

$-a < -\sqrt{a}$

$a > \sqrt{a}, a^2 > a$

$a(a - 1) > 0, a \ge 1$





46. A

Sol. For regular quadrilateral n must be multiple of 4. Perpendiculars dropped from a plane circumcentre to side is always collinear. For one of the side to be diameter $P(E) = \frac{{}^5C_1\,{}^8C_1}{{}^{10}C_3}$ and orthocentre is inside for acute angled triangle $P(E) = 1 - \frac{{}^9C_1\,{}^4C_2}{{}^9C_3} = \frac{5}{14}$

SECTION – D

47. 00021.00

Sol. $\begin{vmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{vmatrix} = -5\ \begin{vmatrix} a & d & 1 \\ b & e & 2 \\ c & f & 3 \end{vmatrix} = 3$

$$2\begin{vmatrix} a & d & 1 \\ b & e & 2 \\ c & f & 3 \end{vmatrix} - 3\begin{vmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{vmatrix} = 21$$

$$\begin{vmatrix} a & d & -1 \\ b & e & 1 \\ c & f & 3 \end{vmatrix} = 21$$

48. 00009.00

Sol. Take $a_n = \frac{1}{(-2)^n t_n}$, on solving, we get $a_n = \frac{a_1}{a_1\left(\frac{2+(-2)^n}{6}\right) + (-2)^{n-1}}$,

for periodicity $a_1\left(\frac{2+(-2)^n}{6}\right) + (-2)^{n-1} = 1$,

for undefined $a_1\left(\frac{2+(-2)^n}{6}\right) + (-2)^{n-1} = 0$,

$\lim_{n\to\infty} a_n = 0$

49. 01600.00

Sol. $x^6 - 2x^3 - 8 = (x - \alpha_1)(x - \alpha_2)\ .....(x - \alpha_6)$

$(2\omega)^6 - 2(2\omega)^3 - 8 = (2\omega - \alpha_1)(2\omega - \alpha_1)\ .....\ (2\omega - \alpha_6)$

$((2\omega^2)^6 - 2(2\omega^2)^3 - 8) = (2\omega^2 - \alpha_1)\ .....\ (2\omega - \alpha_6)$

On multiplying $(40)^2 = g(\alpha_1)g(\alpha_2)\ .....\ g(\alpha_6)$

50. 00062.00

Sol. $\sum_{r=0}^{2014}\sum_{k=0}^{r}(-1)^k(k+1)(k+2)\ {}^{2019}C_{r-k} = \sum_{r=0}^{2014} 2\ {}^{2016}C_r = 2^{2017} - 4034$

51. 00061.25

Sol. $\frac{dx}{dy}e^{-x} - \frac{1}{y}e^{-x} = \frac{1}{y^3}$

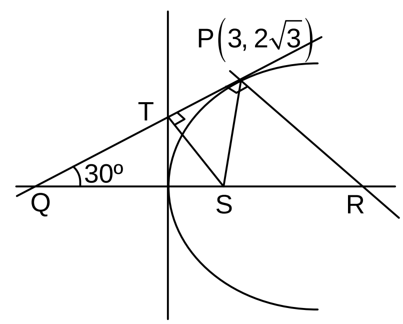
$\frac{d}{dy}\left(e^{-x}y\right) = -\frac{1}{y^2}$, $e^{-x}y = \frac{1}{y} + c$

52. 00018.00

Sol. Area PRST = ΔPQR – ΔQST

$= \frac{1}{2}8\times 2\sqrt{3} - \frac{1}{2}\left(\frac{1}{2}\times 4\times 4\sin 150^{\circ}\right)$

$= 8\sqrt{3} - 2\sqrt{3} = 6\sqrt{3}$



53. 00009.00

Sol. $\int_0^y \sqrt{x^4 + \left(y(3-y)\right)^2}\,dx \le \int_0^y \left(x^2 + y(3-y)\right)dx \le \frac{y^3}{3} + y^2(3-y)$

$f(y) = \frac{y^3}{3} + y^2(3-y)$

$f'(y) = y^2 + 6y - 3y^2 = 2y(3-y) > 0$ for $0 \le y \le 3$ so maximum occurs at y = 3

54. 00056.25

Sol. Given expression is $r_1r_2 + r_2r_3 + r_1r_3 = s^2 = (7.5)^2$