FIITJEE

ALL INDIA TEST SERIES

OPEN TEST

JEE (Advanced)-2019

PAPER – 1

TEST DATE: 03-02-2019

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-D.**

1. ***Section-A (01– 06, 19 – 24, 37 - 42)*** *contains 18 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for all correct answer.*
   *Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.*
   *Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.*
   *Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.*
   *Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).*

   *Negative Marks : **–2** In all other cases.*

   Section-A (07 – 10, 25 – 28, 43 - 46) *contains 12 questions. Based on this section contains* ***TWO (02)*** *paragraphs. Based on each paragraph, there are* ***TWO (02)*** *questions.*
   Each question has ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

2. ***Section-D (11 – 18, 29 – 36, 47 – 54)*** *contains 24 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*

Physics

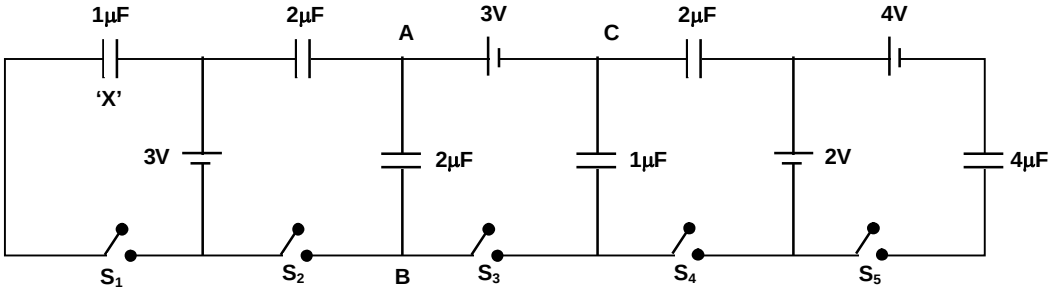
PART – I

SECTION – A
(One or More than one correct type)

This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelength. All the wave lengths are equal or smaller than that of the absorbed photon. Then choose the correct statement(s) (hc = 12400 eV-Å)
   (A) The Ionization energy of the atoms is 489.6 eV.
   (B) The atomic number of atoms is 6.
   (C) The minimum wavelength of emitted radiation is 28.49 Å
   (D) Initially the atom was in first excited state.

2. Six capacitors and four ideal batteries are connected in a circuit as shown in the figure. Initially all capacitors are uncharged and all switches were open. Now, all switches are closed simultaneously. Which of the following statement(s) is/are correct for the circuit shown in figure?



   (A) Work done by 4V battery is 8μJ.
   (B) The potential difference between point A and B is $\frac{19}{7}$V.
   (C) Charge flow by 3V battery from 'C' to 'A' is $\frac{32}{7}$ μC.
   (D) Charge stored in capacitor 'X' is 3 μC.

3. A concave mirror forms real image of a point object 'O' lying on the optical axis at a distance of 60 cm from the pole of mirror. The focal length of mirror is 30 cm. Now, the mirror is cut into two halves symmetrically. In figure-1 both halves are shifted at a distance of 2 mm apart in a direction perpendicular to the optical axis. In figure-2 both halves are rotated (before being separated) about 'O' with reference to original position. Let I_1 and I_2 are the image formed by both halves in both figure. Charge the correct statement(s).

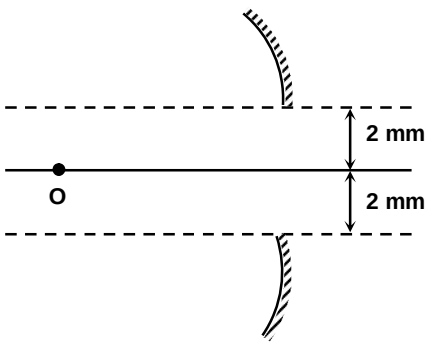


Figure 1

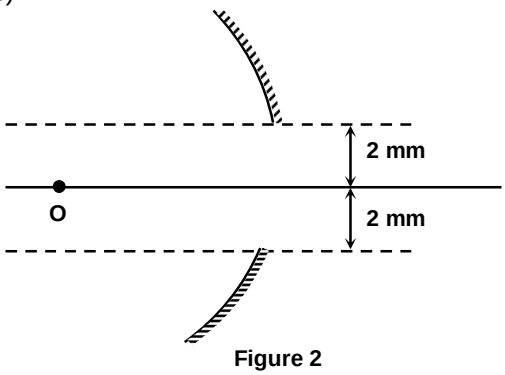
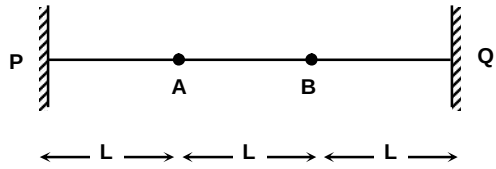


Figure 2

(A) In figure-2 I_1 and I_2 are coincide.
(B) In figure-1 I_1 and I_2 are separated.
(C) In figure-1, the distance between I_1 and I_2 is 8 mm.
(D) In figure-2 the distance between I_1 and I_2 is 4 mm.

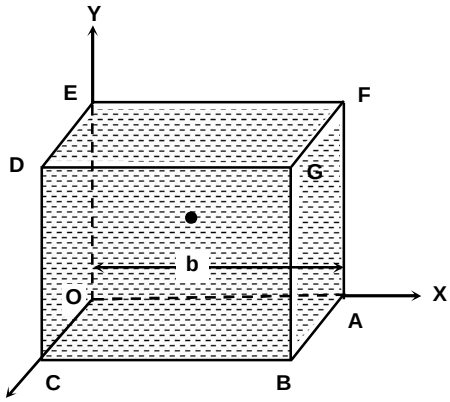
4. A particle is moving in space. Let $\vec{r}$, $\vec{v}$ and $\vec{a}$ are the position vector, velocity vector and acceleration vector of a particle at a given instant. Then
(A) If $\vec{v}\cdot\vec{a}>0$ then magnitude of velocity must be increasing
(B) If $\vec{r}\cdot\vec{v}>0$ then magnitude of position vector must be increasing
(C) If $\vec{r}\cdot\vec{a}>0$ then magnitude of velocity must be increasing
(D) If $\vec{r}\cdot\vec{a}>0$ then magnitude of velocity must be decreasing

5. A string PQ of length 3L is fixed at its both ends. A standing wave is formed in the string with the help of a tuning fork of frequency f_0. The points A and B are node. Regarding the standing wave formed in the string, pick the correct statement(s).



(A) The positions of antinode from the end P are $\frac{L}{2}, \frac{3L}{2}$ and $\frac{5L}{2}$.
(B) The phase difference between two points on the string is either zero or π at a instant $\frac{T}{3}$, where 'T' is the time period of particle.
(C) When particles reach at their extreme positions, then potential energy of the particles is maximum which are near the end A and B.
(D) When particles cross their mean position then kinetic energy of the particles at antinode is maximum.

6. A liquid of density 'ρ' is filled in a cubical container OABCDEFG. The point 'O' is assumed to be origin. A small ball of density ρ_0 is kept at $\left(\frac{b}{2},\frac{b}{2},\frac{b}{2}\right)$ as shown in figure and an acceleration $2g\hat{i}+g\hat{j}$ is given to container. Uniform gravity exists along negative y-axis. Now, choose the correct option(s).

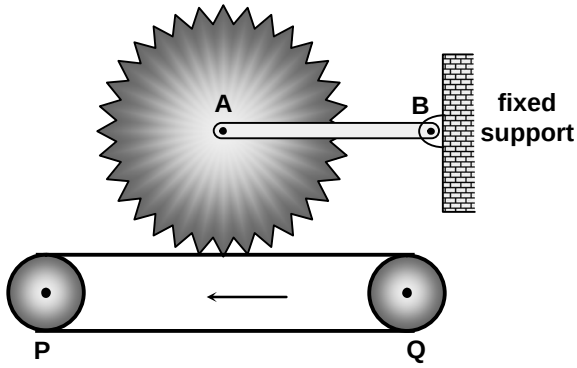


(A) If $\rho = \rho_0$ then acceleration of small ball is $\sqrt{5}g$.
(B) If $\rho = 2\rho_0$ then small ball will hit the mid of edge GF.
(C) If $\rho = 3\rho_0$ then small ball will hit the mid of edge GF.
(D) If $\rho = \rho_0$ then net force on the small ball is zero.

(Paragraph Type)

This section contains **TWO** paragraphs. Based on each paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 7 and 8



A toothed wheel of mass 2 kg and radius $\frac{7}{44}$m is connected with fixed support via a massless rod AB. The total number of teeth on the circumference of wheel is 1000, and they are closely packed. A conveyor belt is kept just below the toothed wheel. Conveyor belt is moving with a constant velocity $\frac{1}{2}$ m/s. P and Q are fixed support for the conveyor belt. The coefficient of friction between teeth of wheel and conveyor belt is 0.4. The moment of inertia of toothed wheel about an axis passing through A and perpendicular to the plane is $\frac{98}{121}$ kg-m^2. Toothed wheel is free to rotate about A and rod AB is also free to rotate about B. Assume normal force acts between teeth and conveyor belt is perpendicular to the surface of conveyor belt. At t = 0, toothed wheel is at rest and brought in contact with the conveyor belt. Now answer the following questions.

7. Total number of teeth that are touched with conveyor belt till the time toothed wheel attain maximum angular velocity is
   (A) 1000
   (B) 1500
   (C) 500
   (D) 250

8. Let 'x' be the distance moved by a particular tooth to attain maximum angular velocity of toothed wheel and 'y' be the distance moved by a fixed point on the conveyor belt for same time then $\frac{y}{x}$ is given by
   (A) 1/4
   (B) 1
   (C) 2
   (D) 1/2

Paragraph for Question Nos. 9 and 10

In a motor cycle engine, when combustion occurs then there is an alternate compression and expansion of the gas product inside the engine of cylinder. In the expansion stroke, the mixture of gaseous product in the cylinder undergoes an adiabatic expansion. Assuming that the gauge pressure immediately after combustion is 20.0 atm in the cylinder, initially volume is 50 cm^3 and the volume of mixture after expansion is 400 cm^3. Assume that adiabatic exponent of gas involved in the cylinder is 4/3.

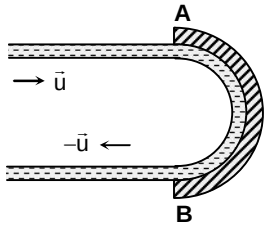
9. Pressure of the engine chamber after expansion is
   (A) 1.2500 atm
   (B) 0.65625 atm
   (C) 1.3125 atm
   (D) 0.62500 atm

10. Work done by the gas during expansion is
(A) 142.5 Joule
(B) 157.5 Joule
(C) 47.50 Joule
(D) 95.0 Joule

SECTION – D
(Numerical Answer Type)

This section contains **EIGHT** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. xxxxx.xx).

11. AB is stationary curved surface blade. A stream line water impinges on the curved surface. The speed of stream line water is 'u', both before and after it strikes the curved surface of the blade. The mass flow rate of water per unit time is 'μ'. Find the force exerted by water on the blade in Newton (Given u = 2.2 m/s, μ = 200 gm/sec)

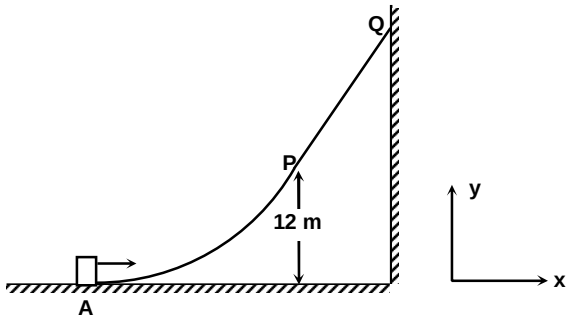


12. A planet is revolving around the sun in elliptical orbit with a semi-major axis of 4×10^8 km. Find the speed of the planet , in km/sec, when it is at a distance of 2×10^8 km from the sun. Given that mass of sun is 2×10^{30} kg and universal gravitational constant is $6.67 \times 10^{-11}\ \frac{N.m^2}{kg^2}$.

13. A vernier calipers is used to measure the diameter of a cylinder. When jaws of vernier are brought close to each other then zero of vernier scale is slightly left to the zero of main scale, and 6^{th} division of vernier scale is exactly coincide with the one of the mark of main scale. Total number of division on vernier scale is 10 which is equivalent to 9 mm. When cylinder is fixed between the jaws then it is found that zero of the vernier scale lies between 6 mm and 7 mm of the main scale and 7^{th} division of vernier scale exactly coincide with one of the main scale division. Find the diameter (in mm) of cylinder.

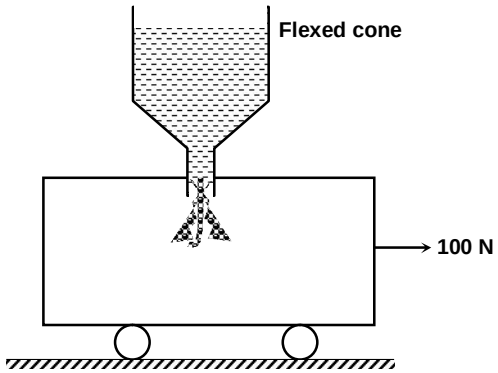
14. A non-conducting solid sphere has a charge density given by $\rho = \rho_0 r^n$, where ρ_0, n are constant and 'r' is the distance from the centre of sphere. If the magnitude of electric field at half of the radius is $\frac{1}{16}$ times that of the surface of non-conductor then find the value of n.

15. APQ is a rough surface. The portion AP is parabolic given by equation $x^2 = 16y$. The point 'A' assumed to be origin. The portion PQ is a straight line of length $20\sqrt{3}$ m. the coefficient of friction between block and surface (APQ) is $\frac{1}{\sqrt{3}}$. A block of mass 3 kg is moved on surface APQ by applying a force on it. The direction of force is always tangential to the surface APQ. The line PQ is tangent on the curve AP at P. If the block moves very slowly on the surface then find the work done by applying force in joule.

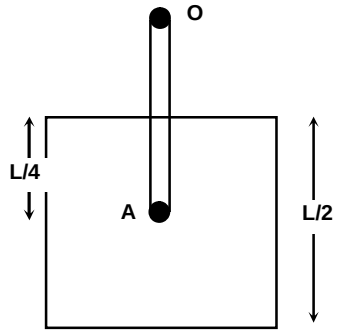


16. A hydrogen like atoms are kept in a closed container. All atoms are at same excited state initially. A photons of energy 3.4 eV are incident on the container then atoms get excited at some higher excited state. During de-excitation of atoms, they emits maximum six types of wavelength of photons, some of them have energy 3.4 eV, some of them have more than 3.4 eV and some less than 3.4 eV. It is assumed that electrons remain in orbit for a finite time during de-excitation. Let a_1 is the maximum acceleration in a given orbit during de-excitation and a_2 is the minimum acceleration in a given orbit. Assuming Bohr's theory is applicable, find ratio of a_1/a_2.

17. A container of mass 200 Kg is moving towards right by applying a constant horizontal force 100 N. Initial velocity of container is zero. Sand is falling in container with a constant rate of 1 kg/sec with the help of a fixed cone as shown in the figure. Find the velocity of the container (in m/s) at t = 100 sec.



18. A rod OA of uniform linear mass density and length 'L' is hinged at O in a vertical plane. Another square plate of same mass is attached with a point A of the rod as shown in the figure. In case (i) square plate is free to rotate about point A and in case (ii) square plate is fixed at point A. Now, system is allowed to oscillate in vertical plane such that axis of rotation is horizontal and passing through point O. Let T_1 and T_2 are the time periods in case (i) and case (ii) respectively. If T_1/T_2 is $m\sqrt{\frac{6}{11}}$. Find the value of m.



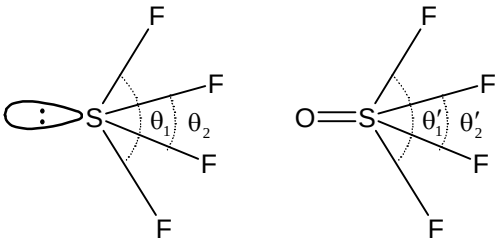
Chemistry

PART – II

SECTION – A
(One or More than one correct type)

This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

19. For the following geometry, which of the inequality is/are correct?



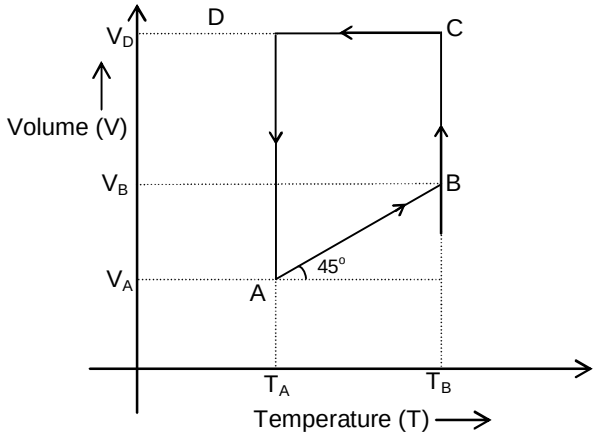
(A) $\theta_1' < \theta_1$

(B) $\theta_2 < \theta_2'$

(C) $\theta_1 < \theta_1'$

(D) $\theta_2 > \theta_2'$

20. Two moles of a monoatomic ideal gas is taken through a reversible cyclic process starting from "A" as shown in figure.



The volume ratio $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If temperature T_A at 'A' is 27^oC, then choose the correct option(s)

(A) Temperature of the gas at point 'B' is 600 K.

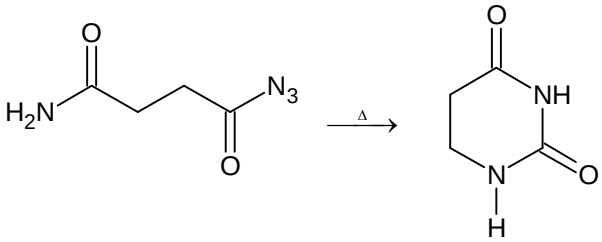
(B) Work done in cyclic process (ABCDA) is -1200 cal.

(C) Work done in the process (C → B) is – 1663.68 cal.

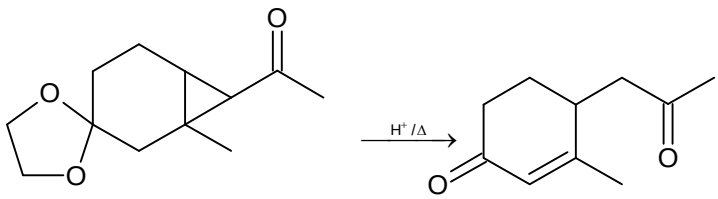
(D) Work done in the process (A → D) is – 1663.68 cal.

21. Choose the reaction(s) with correct major products:

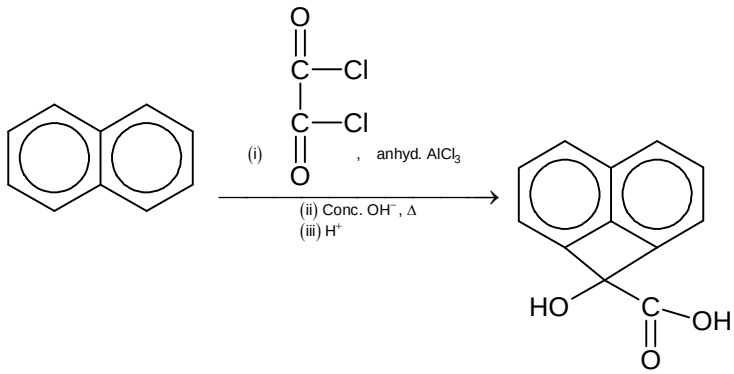
(A)



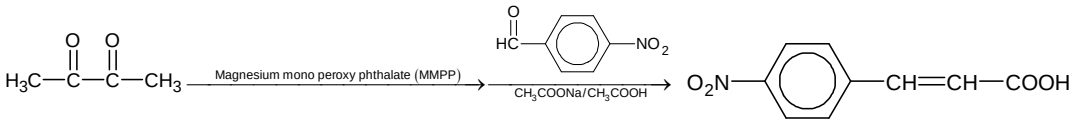
(B)



(C)

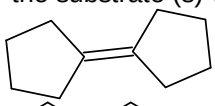


(D)

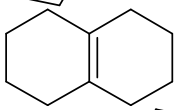


22. Identify the substrate (s) which on acidic hydrolysis produces the same major product?

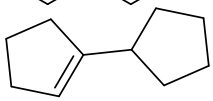
(A)



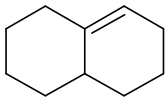
(B)



(C)



(D)



23. Choose the only **incorrect** statement(s) among the following

(A) All boron halides are ionic halides.
(B) Anhydrous aluminium chloride is an ionic compound.
(C) Coordination number of Al in $AlCl_3$(s) is 4.
(D) BI_3 is the least acidic among all boron halides.

24. For which of the following complexes, IUPAC names is/are correctly matched?

(A) $Na\left[Pt(NH_3)BrCl(NO_2)\right]$-Sodiumamminebromidochloridonitrito-N-platinate (II)

(B) $\left[CuCl_2\left\{O=C(NH_2)_2\right\}_2\right]$ – Dichloridobis(dicarboxidotetraamine)copper(II)

(C) $\left[PtCl_3\left(\eta^2-C_2H_4\right)\right]^-$ -Trichlorido$\left(\eta^2-\text{ethene}\right)$platinate(II) ion.

(D) $\left[FeCl(CN)_4(O_2)\right]^{4-}$ -Chloridotetracyanosuperoxoferrate(II) ion

(Paragraph Type)

This section contains **TWO** paragraphs. Based on each paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 25 and 26

The periodic system of the elements in our three dimensional world is based on the four quantum numbers n = 1, 2, 3,……;

$\ell = 0,1,$ ……n -1; $m_\ell = 0, \pm1, \pm2 \ldots\ldots \pm\ell$; and $m_s = \pm\frac{1}{2}$

Let us move to flatlandia. It is a two dimensional world where the periodic system of elements is based on three quantum numbers.

n = 1, 2, 3……; m = 0, $\pm1, \pm2, \pm3, \ldots\ldots\ldots \pm(n-1)$; and $m_s = \pm\frac{1}{2}$. 'm' plays the combined role of ℓ and m_ℓ of the three dimensional world (for example s, p, d….level are related to 'm'). Some tasks and basic principles, related to this two dimensional flatlandia where the chemical and physical experience obtained from our common three dimensional world, is applicable. Assume Auf-Bau rule, Hund's rule of maximum multiplicity and Pauli's exclusion principle which are obeyed in three dimensional world, are also obeyed in 2-dimensional world as such. Energy order of various orbitals for filling up the electrons in multi electron atoms in flatlandia is governed by (n + |m|) rule which is analogous to the (n + l) rule in three dimensional world.
Now, answer the following questions:

25. Total number of elements in the first four periods of flatlandia periodic table in two dimensional world will be:
(A) 36
(B) 27
(C) 24
(D) 18

26. Identify the only **incorrect** statement among the following: ('n' represents valence shell and 'z' represents atomic number)
(A) In flatlandia, the maximum covalency of an element with atomic number 5 is 3.
(B) sextet rule corresponds to the octet rule in flatlandia.
(C) Total number of lone pair of electrons in all the simple binary compounds of the elements (n = 2) with the lightest element (z = 1) are four.
(D) In flatlandia, if elements with atomic number 1 and 5 are denoted by H and C respectively then the number of π-bonds in C_2H_2 molecule is 1.

Paragraph for Question Nos. 27 and 28

A white crystalline solid (X) exhibits the following observations:
(i) The flame of bunsen burner is intensely yellow coloured.
(ii) An aqueous solution is neutral; dropwise addition of sulphurous acid (an SO_2 solution) leads to a deep brown solution which is decolourised in the presence of excess of sulphurous acid.

(iii) If $AgNO_3$ solution is added to the discoloured solution in (ii) and acidified with HNO_3, a yellow precipitate that is insoluble on addition of NH_3, but that can be readily dissolved by adding CN^- or $S_2O_3^{2-}$, is obtained.

(iv) If an aqueous solution of the solid (X) is treated with KI and dilute H_2SO_4, a deep brown solution is formed that can be decolourised by addition of sulphurous acid or $Na_2S_2O_3$ solution.

(v) An amount of 0.1000 g of the solid (X) is dissolved in water, 0.5g KI and a few mL of dilute H_2SO_4 are added. The deep brown solution formed is titrated with a 0.1000 M $Na_2S_2O_3$ solution until the solution is completely decolourised; the consumption, 37.40 mL. (Atomic mass : Na = 23, K = 39, Cl = 35.5, I = 127, O = 16)
Based on the above paragraph, now answer the following questions:

27. % by mass of 'Cl' in the solid (X) is
(A) 59.34%
(B) 29%
(C) 40.27%
(D) 0 %

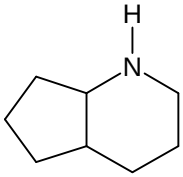
28. On the basis of the observations given in the above paragraph, solid (X) can be:
(A) NaI_3
(B) $NaClO_3$
(C) $NaIO_4$
(D) $NaClO_4$

SECTION – D
(Numerical Answer Type)

This section contains **EIGHT** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. xxxxx.xx).

29. When light of suitable frequency is incident on a mixture of gaseous chlorine and hydrogen, hydrogen chloride is formed. The mixture is irradiated with a mercury UV-lamp (λ = 253.6 nm). The lamp has a power input of 10 watt. An amount of 2% of the energy supplied is absorbed by the gas mixture (in a 10 litre vessel). Within 2.5 seconds of irradiation, 65 millimoles of HCl is formed. If the quantum efficiency of this reaction is found to be $x \times 10^4$, then find th value of 'x'.
(Quantum efficiency is defined as the number of product molecules formed per absorbed photon)
(Take Avogadro's number = 6.02×10^{23}, Plank's constant = 6.6×10^{-34} Js)

30. An organic compound (P) has molecular formula, $C_8H_{11}NO$ and it can be resolved into enantiomers. (P) does not decolourise bromine water solution. 'P' on refluxing with dilute H_2SO_4 yields another resolvable compound Q ($C_8H_{12}O_3$) which give effervescence with $NaHCO_3$. 'Q' on treatment with $NaBH_4$ yields R($C_8H_{14}O_3$) which on heating with conc. H_2SO_4 yields S($C_8H_{12}O_2$). Compound (P) on reduction with $LiAlH_4$, followed by treatment of H_2SO_4 yields the compound which is given below



Let the sum of degree of unsaturation in P, Q, R and S be 'x'; the sum of the number of rings in P, Q, R and S be 'y' and the sum of distinct functional groups in P, Q, R and S be 'z' (if the same functional group appears in two or more compounds count it once only).

Then find, the value of $\frac{x+y+z}{y+z}$.

31. For an ideal gas, temperature varies as:
$T = K + \beta V^2$
Where K and β are positive constants and 'V' is the molar volume of the gas, if the minimum pressure (P_{min}) of one mole of an ideal gas is expressed as:
$P_{min} = nR^a\beta^bK^c$
Then find the value of $\frac{n+a+b}{n-a+b-c}$

32. If 'θ' is the angle between two bonds on the same central atom, then fraction of s-character of the bond can be calculated as:
$\cos\theta = \frac{s}{s-1}$ (where $90^o < \theta < 180^o$)
Let fraction of s-character of each orbital having lone pair in water be 'x' and fraction of s-character of the lone pair in NH_3 be 'y', then find the value of (x + y).
(cos 104.5^o = - 0.25 and cos 107^o = - 0.30)

33. Some aqueous solutions are given below. If 'x' be the number of solution(s) whose pH does not change appreciably by adding small amount of H_2SO_4 and 'y' be the number of solution(s) whose pH changes rapidly by adding H_2SO_4, then find the value of $\frac{y}{x}$.

(a) 0.1 M NH_4CN

(b) 0.1 M NH_4OH + 0.1 M NH_4Cl

(c) 100 mL, 0.1 M HCl + 100 mL, 0.5 M $Ca(OH)_2$

(d) 100 mL, 0.1 M HCl + 100 mL, 0.2 M NaCN

(e) 100 mL, 0.1 M $Ca(OH)_2$ + 100 mL, 0.4 M H_2S

(f) 100 mL, 0.1 M NaOH + 100 mL, 0.2 M HCl

(g) 100 mL, 0.1 M NaOH + 100 mL, 0.2 M K_2SO_4

(h) 100 mL, 0.5 M CH_3COOH + 100 mL, 0.1 M $Ba(OH)_2$

(i) 100 mL, 0.5 M $Ca(OH)_2$ + 100 mL, 0.1 M HCl

34. Consider the following ions:
$S_2O_2^{2-}; S_2O_3^{2-}; S_2O_4^{2-}; S_2O_5^{2-}; S_2O_6^{2-}; S_2O_7^{2-}; S_2O_8^{2-};$
If the number of ions having S – S or S = S linkage be 'x' and the number of ions having S – O – S linkage be 'y', the number of ions having peroxy linkage b ' z',. Find the value of $\left(\frac{x}{y+z}\right)$.

35. o-aminobenzaldehyde is heated with acetone in dilute NaOH solution to produce the major organic product (A), which on treatment with catalytic amount of H_2SO_4, produces another major organic product (B).
Let, the degree of unsaturation of (A) be 'x'; the degree of unsaturation of (B) be 'y'; the number of rings in (A) be 'z' and the number of rings in (B) be 'p', then, find the value of $\left(\frac{y+p}{x-z}\right)$.

36. How many milliliters of 2.0 M NaOH solution must be added to 100 mL of 0.1 M H_3PO_4 solution in order to obtain a buffer solution with pH about 7.2?
(The pK_a values of H_3PO_4 are $pK_{a_1} = 2.1;\ pK_{a_2} = 7.2$ and $pK_{a_3} = 12.0$)
Assume the temperature of reaction mixture to be 298 K.

Mathematics

PART – III

SECTION – A
(One or More than one correct type)

This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. Let (x, y) be the ordered pair satisfying the inequalities $\sin\frac{\pi x}{2}+\cos\pi y \geq 2$ and x > 1, where x, $y \in \mathbf{N}$. Choose the **correct** statement(s).
(A) if $x^y - y^x$ is divisible by 8 remainder can be 1.
(B) if $x^y + y^x$ is divided by 8 remainder can be 1.
(C) if $x^y + y^x$ is divided by 8 remainder can be 3.
(D) if $x^y - y^x$ is divided by 8 remainder can be 5.

38. f : $\mathbf{R} \to \mathbf{R}$, be a twice differentiable function such that $f(x)f''(x) \neq 0$, $\forall\ x \in \mathbf{R}$. Choose the **correct** option(s).
(A) $y = f(x)f'(x)$ is a strictly increasing function.
(B) if $f'(x) = 0$ for some $x \in \mathbf{R}$, then it is always point of inflection.
(C) $f(x)f''(x) > 0\ \forall\ x \in \mathbf{R}$
(D) if $f'(x_0) = 0$ and $f(x_0) < 0$ then it is point of local maxima.

39. The roots of the equation $az^4 + bz^2 + cz + d = 0$ are vertices of a convex quadrilateral in argand plane (a, b, c, d ∈ C). Choose the **correct** option(s).
(A) The intersection point of diagonals is necessarily origin.
(B) If the quadrilateral is a parallelogram then point of intersection of diagonals is origin.
(C) If the quadrilateral is a rhombus then it is necessarily a square.
(D) If the quadrilateral is rhombus then its area is $2\sqrt{\left|\frac{d}{a}\right|}$.

40. Let $\mathbf{N} \cup \{0\} \to \mathbf{N} \cup \{0\}$, $f_k(x)$ be a function such that $f_k(x)$ is equal to remainder when x^k is divided by 10. ($k \in N$) choose the **incorrect** option(s).
(A) $y = f_k(x)$ is periodic function with period 3.
(B) $f_{2019}(f_{2017}(x) - x) = f_{2019}(f_{2013}(x) - x)$ has infinitely many solutions.
(C) $f_{2019}(f_{2018}(f_{2017}(..... f_1(x)))) = x$ is only true when x has 0, 1, 5 at unit place.
(D) $y = f_k(x)$ is non periodic function.

41. Let $f(x) = e^{x+1} - 1$, $g(x) = p|f(x)| - \sum_{k=1}^{n}\left|f\left(x^k\right)\right|$, ($n \in \mathbf{N}$). It is given that g(x) is differentiable function over the real numbers. Choose the **correct** option(s).
(A) p can assume the value 99.
(B) if p is 100 then n can have two different values.
(C) if p is 100 then sum of possible values of m is 39.
(D) p can assume the value 36.

42. The line y = t intersects $y = x^3 - 3x + 2$ at three points $(f_1(t), t)(f_2(t), t)(f_3(t), t)$ such that $f_3(t) > f_2(t) > f_1(t)$. Define a function $g(t) = t(f_3(t) - f_1(t))$. Choose the **correct** option(s).
(A) $g(2) = 4\sqrt{3}$
(B) $g(2) = \frac{2}{3}$
(C) $g'(2) = 2\sqrt{3}$
(D) $g'(2) = -\frac{2}{3}$

(Paragraph Type)

This section contains **TWO** paragraphs. Based on each paragraph, there are **TWO** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

Paragraph for Question Nos. 43 and 44

Let P be a point in X-Y plane and P′ is a point such that $OP \cdot OP' = r^2$, where O, P and P′ are collinear and O divides P and P′ externally. (O is the origin)

43. If point P lies on the circle $(x-1)^2 + (y-5)^2 = 4$ and P′ also satisfy the same circle, then value of r^2 is
(A) 30
(B) 22
(C) 4
(D) none of these

44. If the point P lies on the line $x + y = 1$, then P′ traces
(A) A straight line of slope $\frac{1}{r}$
(B) A circle of radius $\frac{r}{\sqrt{2}}$
(C) A circle of radius $\frac{r^2}{\sqrt{2}}$
(D) A circle of radius $\frac{r^2}{2}$

Paragraph for Question Nos. 45 and 46

Let $y = f(x)$ be an odd cubic polynomial function such that $f(x) = f(\alpha)$ has all the roots real and $y = |f(x) - f(\alpha)|$ is non differentiable at $x = \alpha$ only. Let $y = g(x)$ be another cubic polynomial such that $y = f(x) - g(x)$ is quadratic expression and $|g(x)| \le M$, $\forall\ |x| \le \alpha$, then minimum value of M is $f(\alpha)$.

45. If $f'''(0) = 6$, and $\alpha = 2$, then f(x) is equal to
(A) $x^3 + 2x$
(B) $x^3 - 2x$
(C) $x^3 + 3x$
(D) $x^3 - 3x$

46. If $|4x^3 + ax^2 + bx + c| \le M$ for $\forall\ |x| \le 1$, then minimum value of 4M is
(A) 4
(B) 1
(C) 2
(D) 3

SECTION – D
(Numerical Answer Type)

This section contains **EIGHT** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. xxxxx.xx).

47. Find the minimum value of $\sum_{k=0}^{4}\sum_{r=0}^{2019}\left|z - \frac{e^{i\left(\frac{k}{3}+\frac{r}{2}\right)\pi}}{1010}\right|$.

48. ABCD is a regular tetrahedron with side length $\left(\sqrt{2}+1\right)$ units. A point P is taken on the edge AB such that $\frac{AP}{BP}=\sqrt{2}-1$. Q is the point in the plane of triangle BCD such that PQ $\perp$ AB. If d is perpendicular distance of P from plane BCD and d_1 is minimum distance between P and Q, then find the value of $\left(\frac{d_1}{d}\right)^4$.

49. f : R $\to$ R is a differentiable function satisfying $2(f(x))^2f'(x) = (f(2x + 1))^2f'(2x + 1)$, $f\left(-\frac{1}{8}\right)=1$, $f(6) = 2$ find the value of $\frac{2}{\left(f\left(-1\right)\right)^3}$.

50. y = f(x) is a parabola having $\left(0,\frac{3}{2}\right)$ and (0, 0) as vertex and focus respectively, find the number of roots of the equation $\frac{1}{2f\left(x\right)-3}-\frac{1}{2f\left(x\right)+3}+\frac{6}{x^2}=0$.

51. Out of 10 tokens, numbered from {1, 2, ..., 10}, first person X chooses a token then person Y chooses the token (without replacement in both cases). A person is winner if he gets a prime number and in the next draw other person get a non prime. If both gets prime in successive draw, game is a draw. Game continuous till some one wins or game is drawn. If P(x) is probability of X winning, P(y) is probability of Y wining and P(d) is probability of draw, then the value of $21P(x) + 126P(y) - 7P(d)$ is _____

52. A point $P\left(\frac{e^t+e^{-t}}{2},\frac{e^t-e^{-t}}{2}\right)$ traces a locus S = 0 in X–Y plane (t being a non negative parameter). A fixed point P_0, having parameter t_0, lies on the curve S = 0. The area bounded by the curve, line OP_0 and x-axis in 240 sq. units (O being the origin), find the value of t_0.

53. Given that $\cos\alpha\cos\beta + \sin^2\alpha\sin\beta - \cos^2\gamma\sin\alpha = 1$, $0 < \alpha, \beta, \gamma < \pi$.
Find the value of $\sin(\alpha + \beta - \gamma) + \sin(\beta + \gamma - \alpha) + \sin(\alpha + \gamma - \beta)$.

54. Ten points are given in a plane such that no three points are collinear and no four of them form a trapezium. From every point, all the possible lines are drawn which are parallel to lines formed by taking any two points out of remaining points. Find the total number of points of intersection of these lines. (excluding the given ten points)