FIITJEE

ALL INDIA TEST SERIES

FULL TEST – VIII

JEE (Main)-2019

TEST DATE: 31-03-2019

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. D

Sol. $\frac{dN}{dt} = R - \lambda N,$

Solving,

$N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$

If the process mentioned in option (D) is performed a large no. of times and the activity when introducing into the reactor is A_0, then:

Just after withdrawing from reactor, the activity is:

$A = \frac{A_0}{2} + \frac{R}{2}$

and after one $\frac{1}{2}$ – life outside it should be A_0:

$A_0 = \frac{A_0}{4} + \frac{R}{4}$ or $A_0 = \frac{R}{3}$.

2. C

Sol. For large values of x, $\left[1 - e^{-kx^2}\right] \to 1$

For small values of x, $\left[1 - e^{-kx^2}\right] \approx kx^2,$

Thus for large x (and ‘a’) it is like SHM; for small values the time period can be determined from dimensional analysis:

3. B

Sol. It is clear that there is a pressure antinode at the wall and there is pressure node at P when it is silent (no sound).

So, $L_0 = (2m+1)\frac{\lambda}{4}$, $m = \text{integer}$ and λ = wavelength of sound.

Further: $L_0 - \frac{v_0 t_0}{2} = n\frac{\lambda}{2}$, $n = \text{integer}$.

Taking the difference and considering different values of m, n we arrive at the conclusion.

4. B

Sol. As the radius of the container is finite, the viscous force becomes larger then that predicted by Stokes' Law.

5. D

Sol. The stress is compressive on the left end, tensile on the right.

6. D

Sol. When the slits have unequal widths, maximum intensity = $k'(A_1 + A_2)^2$

minimum intensity = $k'(A_1 - A_2)^2$

Where A_1, A_2 are individual amplitudes.

When the length of the slits is decreased, there is less light at the maxima and the fringes become curved.

7. A

Sol. The reading on C_1 is = $2.5 + 6 \times \left(1 - \frac{23}{25}\right) \times 0.1$ = 2.548 cm and the reading on

C_2 is = $2.6 + 6 \times \left(1 - \frac{27}{25}\right) \times 0.1$ = 2.552 cm.

8. C

Sol. $f_0 = \frac{8}{2L}\sqrt{\frac{T_1}{\mu}} = \frac{10}{2L}\sqrt{\frac{T_2}{\mu}}$

$\frac{T_2}{T_1} = \left(\frac{8}{10}\right)^2 = \frac{(\rho_s - \rho_w)V_g}{\rho_s V_g}$

$\therefore \quad \frac{\rho_s}{\rho_w} = 2.8$.

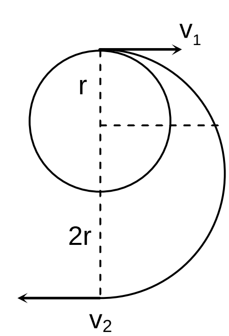
9. B

Sol. Applying conservation of Angular momentum $mv_1.r = mv_2.2r$

Applying conservation of energy, $\frac{1}{2}mv_1^2 - \frac{GMm}{r} = \frac{1}{2}mv_2^2 - \frac{GM}{2r}m$.

Solving, $v_1 = \frac{2v_0}{\sqrt{3}}$.

$\therefore$ The impulse per unit mass = $\frac{mv_1 - mv_0}{m}$.



10. D

Sol. The object and image positions relative to their closest lenses do not depend on the separation between the two lenses. So the rays between the lenses are parallel. Object is at the focus of the first lens and image at the appropriate focus of the second lens.

11. C

Sol. $BP - AP \approx \frac{d^2}{2D} = \frac{\lambda}{4}$ (given)

$CP - AP \approx \frac{4d^2}{2D} = \lambda$

Resultant amplitude at P is A′, where $A'^2 = (A + A)^2 + A^2$

12. B

Sol. $\frac{mv^2}{R} = evB$ …(1)

$mvR = \frac{nh}{2\pi}$ …(2)

Dividing (2) by (1), we get the result.

13. C

Sol. $KE_{max} = h\nu - W = 5.93 - 1.9 = 4\,eV$ nearly.

14. D

Sol. The flux of the magnetic field through the loop when it has rotated through $\theta = \omega t$, is

$\Phi = \frac{B\pi R^2}{2}(2\cos 45^\circ \cos\omega t)$

$EMF = -\frac{d\Phi}{dt} = \frac{B\pi R^2\omega}{\sqrt{2}}\sin\omega t$

Current, $i = \frac{EMF}{\lambda.2\pi R}$

Magnetic moment = current × area

$= \left(\frac{B\pi R^3\omega}{4\lambda}\sin\omega t\right)$

15. B

Sol. $EMF \simeq \frac{\mu_0 iN}{2\pi}\left[\frac{1}{a_0} - \frac{1}{a_0 + b}\right]\ell v,$

$v = \omega A \sin\omega t$.

16. B

Sol. $EMF = \frac{d}{dt}(Bvt \times vt) = 2Bv^2t$

and resistance = $\lambda\left(2vt + 2\sqrt{2}vt\right)$

17. A
Sol. Change in flux (linked)

$$\Delta\Phi = 120 \times \mu_0 ni.\pi\left(\frac{2}{100}\right)^2$$

Charge that flows through coil = $\frac{\Delta\Phi}{R}$, R = 3.6 Ω.

18. A
Sol. $V = k\int_{x=r}^{R} \frac{\sigma.2\pi x dx}{x} = k.\sigma 2\pi(R-r)$

19. D
Sol. $q = \frac{C_0C'}{C_0 + C'}E$, where E = 6V

$$\frac{dq}{dt} = \frac{C_0}{C_0 + C'}E\frac{dC'}{dt} - \frac{C_0C'}{(C_0 + C')^2}E\frac{dC'}{dt}$$

$$= \left(\frac{C_0}{C_0 + C'}\right)^2 E\frac{dC'}{dt}$$

Where $C_0 = 2\mu F$, $\frac{dC'}{dt} = 2\mu F/s$.

The value of C′ can be found to be 4 μF at the given instant.

20. D
Sol. For an elastic collision between a mass m_1 and a second mass m_2, at rest, we get the final velocities:

$$v_2 = \frac{2m_1}{m_1 + m_2}v_0$$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}v_0$$

When v_0 is speed of approach of m_1.

21. D
Sol. The three horizontal tensions at the lowest points are clearly equal to T_0, say. Then for chain A :

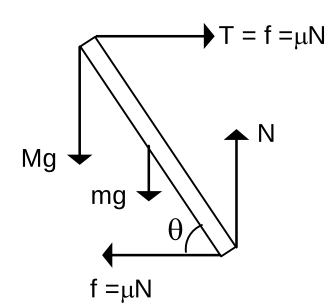
$$T_A \cos\theta_A = m_A g$$

$$T_A \sin\theta_A = T_0$$

i.e. $m_A g = T_0 \cot\theta_A$

22. A
Sol. If the net torque about the lowest point is set equal to zero, we get, $MgL\cos\theta + mg\frac{L}{2}\cos\theta - \mu N.L\sin\theta = 0$

Where N = (M + m)g.



23. B

Sol. the overall modulation index is found from the expression: $m^2 = m_1^2 + m_2^2$

Where $m_1 = 0.6$ and $m_2 = 0.7$.

24. C

Sol. Since some of the molecules have dissociated, we must take into account the energy required for dissociating the bonds into account.

$U_i^{kin} = \left(N - \frac{N}{3}\right)\frac{5}{2}RT + \frac{N}{3} \times 2 \times \frac{3RT}{2}$

By 1st Law,

$Q = \Delta Q = \Delta U = \Delta U^{kin} + \Delta U^{bond}$.

25. D

Sol. The final C.M. is at $\frac{H}{8}$ above the base. The volume, and consequently the mass of a cone (of fixed vertical angle) is proportional to the cube of its height. Taking the origin at apex.

$\frac{7H}{8} = \frac{H^3 \frac{3H}{4} - x^3 \frac{3x}{4}}{H^3 - x^3}$

Rearranging, we get the required condition.

26. C

Sol. $\frac{1}{2}v'^2 = \left(\frac{GM}{2R^2}\right).R = \frac{GM}{2R} = \frac{1}{4}.\frac{2GM}{R} = \frac{1}{4}.v_0^2$

27. C

Sol. $C_{v,mix} = \frac{n_1 C_{v_1} + n_2 Cv_2}{n_1 + n_2}$

$C_{p,\,mix} = C_{v,mix} + R$

$\gamma_{mix} = \frac{C_{p,mix}}{C_{v,mix}}$

28. B

Sol. For the diffraction pattern, (1μm) sin 30° = 1λ

For the interference pattern, dsin30° = 10λ, since the 10th maximum falls on the diffraction minimum.

29. C

Sol. For Carnot engines, efficiency = $1 - \frac{T_2}{T_1} = \frac{1}{5}$, in this case.

Let the intermediate temperature be T′.

Then $1 - \frac{T_2}{T'} = \frac{1}{11}$

Solving, and calculating $e = 1 - \frac{T'}{T_1}$

We get 12%.

30. C

Sol. $i_1 = \dfrac{e_0}{\sqrt{R^2 + \left(\dfrac{1}{\omega_1 C}\right)^2}} = \dfrac{e_0}{Z_1}$, where $\omega_1 = 100\pi$

$i_2 = \dfrac{e_0}{\sqrt{R^2 + \left(\dfrac{1}{\omega_2 C}\right)^2}} = \dfrac{e_0}{Z_2}$, where $\omega_2 = 500\pi$

So, $Z_1 > Z_2$, therefore $i_1 < i_2$.

Chemistry

PART – II

SECTION – A

31. D

Sol. $d = \frac{PM}{RT}$

$M_{(\text{dry air})} = \frac{(28 \times 0.8) + (32 \times 0.2)}{1} = 28.8\,\text{gm/mol}$

$d_{(\text{dry air})} = \frac{P \times 28.8}{RT}$ … (i)

$R.H. = \frac{P_{H_2O}}{\text{Vapour pressure}} \times 100$

$40 = \frac{P_{H_2O}}{19} \times 100$

$P_{H_2O} = \frac{19}{2.5}$ torr

% of H_2O(vapours) in air $= \frac{19/2.5}{760/1} \times 100 = 1\%$

M_{moist} air = (18 × 0.01) + (28.8 × 0.99)
= 28.692

$d_{\text{moist air}} = \frac{P \times (28.692)}{RT}$ … (ii)

$d_{\text{dry air}} - d_{\text{moist air}} = \frac{P}{RT}(28.8 - 28.692)$

$= \frac{12}{300} \times 0.108$

$= 4.38 \times 10^{-3}$ g/L

32. C

Sol. (A) $HCHO + 50\%\ \text{conc.}NaOH \longrightarrow HCOO^-Na^+ + CH_3OH$

(B) $P_4 + 3NaOH + 3H_2O \longrightarrow 3NaH_2PO_2 + PH_3 \uparrow$

(C) $5Br^- + BrO_3^- + 6H^+ \longrightarrow 3Br_2 + 3H_2O$

(D) $H_2O_2 \xrightarrow{h\nu} H_2O + \frac{1}{2}O_2$

33. B

Sol. $W = -2.303 \times 2 \times 8.314 \times 300 \log\frac{0.6}{1.2}$

$= 3.48$ kJ

34. D

Sol. For H_2O and $(CH_3)_3COH \Rightarrow$ +ve deviation.

$HgI_2 + 2KI \longrightarrow K_2[HgI_4]$

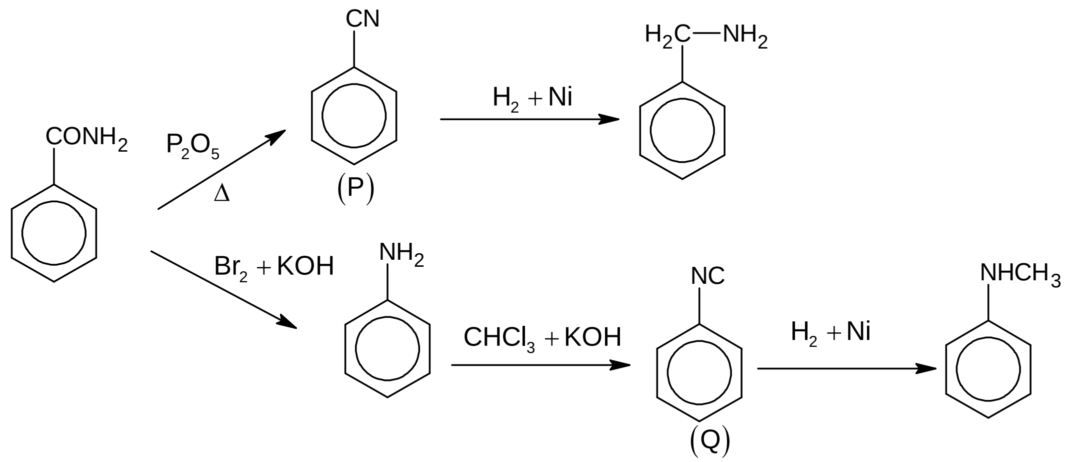
35. C

Sol. (iii) $Na_2S_2O_3 + HCl(aq) \longrightarrow S\downarrow + SO_2 + H_2O + NaCl$

(iv) $H_2S(aq) + SO_2(g) \longrightarrow H_2O + S\downarrow$

36. A

Sol.



37. C

Sol. D – II (meso compound)
C – III
A – I
B – IV

38. C

Sol. $H_2C_2O_4$, NaCl, HCOOH, Na_2CO_3, $K_4[Fe(CN)_6]$

39. C

40. D

41. C

Sol. $BaF_2(s) \longrightarrow Ba^{+2}(aq) + 2F^-(aq)$ ['s' be the solubility of BaF_2]

s 2s

$Ba^{+2} + C_2O_4^{-2} \longrightarrow BaC_2O_4(s)$ $K_{eq} = 10^{-7}$ [Let y mol / lit. of Ba^{+2} is left after reaching equilbrium]

y 0.1 – s

$y(0.1 - s) = 10^{-7}$ …(i)

$y(2s)^2 = 10^{-6}$ …(ii)

By solving s = 0.096 M

$[C_2O_4^{2-}] = 0.1 - 0.096 = 4 \times 10^{-3}$ M

$[F^-] = 2s = 2 \times 0.096 = 0.192$ M

$[Ba^{+2}] = y = 2.7 \times 10^{-5}$ M

42. A
Sol. Haematite – Fe
Galena – Pb
Cassiterite – Sn
Zinc blende – Zn
Only these will involve carbon reduction.

43. D
Sol. A, B, C are wrong.
(C) – LiCl < RbCl < KCl < NaCl

44. A
Sol.
$$2Zn(ClO_3)_2 \xrightarrow{\Delta} 2ZnO + 2Cl_2\uparrow + 5O_2\uparrow$$
$$NH_4ClO_4 \xrightarrow{\Delta} N_2\uparrow + O_2\uparrow + Cl_2\uparrow + H_2O\uparrow$$
$$K_2Cr_2O_7 \xrightarrow{\Delta} K_2CrO_4 + Cr_2O_3 + O_2\uparrow$$
$$KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2\uparrow$$

45. B
Sol.

	A	+	$2B \longrightarrow$ Product
t=0	a		a
$t = t,$	$(a - x)$		$(a - 2x)$

$$[A]_t = ae^{-kt}$$
$$a - x = ae^{-kt}$$
$$x = a(1 - e^{-kt})$$
At $t = t_{1/2} \Rightarrow a - x = \frac{a}{2}$
$$x = \frac{a}{2}$$
$$[B]_t = a - 2x$$
$$= a - 2a(1 - e^{-kt})$$
$$= a(2e^{-kt} - 1)$$
At t = 0, [B] = a
$t = t_{1/2}, [B]_t = a - 2x$
= a – a
= 0

46. A
Sol. As particles size increases, the effect of impact average out and the Brownian movement become slow.

47. D
Sol.
$$NO_2^- / NO_3^- \xrightarrow{Zn/NaOH} NH_3 + H_2O$$
$$CaCN_2 + 3H_2O \longrightarrow CaCO_3 + 2NH_3$$
$$NH_4Cl + Ca(OH)_2 \longrightarrow 2NH_3 + CaCl_2 + 2H_2O$$

48. D
Sol. It is a molten mixture of alumina, fluorspar and cryolite.

49. D
Sol.

$$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \xrightarrow{\text{Fusion}} Na_2\overset{+6}{Cr}O_4 [X]$$

$$\downarrow H_3O^+$$

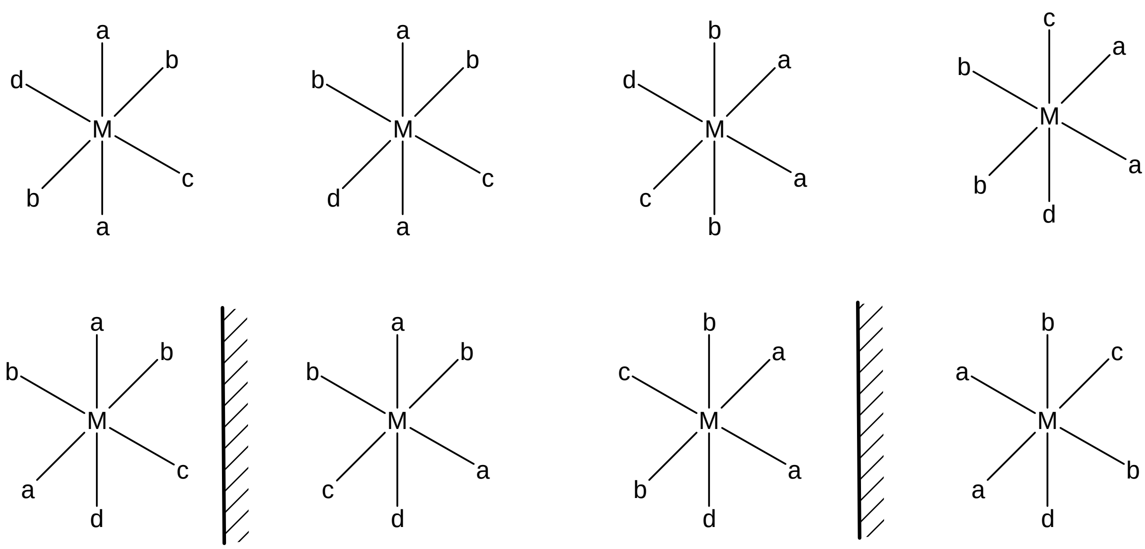
$$Cr^{+3} + O_2 + H_2O \xleftarrow{H_2O} \underset{\substack{\text{(Deep blue violet)} \\ [Z]}}{\overset{+6}{Cr}O(O_2)_2} \xleftarrow{H^+/H_2O_2} Na_2\overset{+6}{Cr}_2O_7 [Y]$$

$$\underset{(Y)}{Na_2Cr_2O_7} + H_2SO_4 \longrightarrow \underset{\text{(Chromic anhydride)}}{2CrO_3} + Na_2SO_4 + H_2O$$

50. C
Sol. (i) $[Cr(H_2O)_6]Cl_3 + AgNO_3 \longrightarrow 3AgCl + ...$
$200 \times 0.01 \times 3 = 0.1 \times V$
$V = 60$ ml
(ii) $[Cr(H_2O)_5Cl]Cl_2.H_2O + 2AgNO_3 \longrightarrow 2AgCl + ...$
$200 \times 0.01 \times 2 = 0.1 \times V$
$V = 40$ ml
(iii) $[Cr(H_2O)_4Cl_2]Cl.2H_2O + AgNO_3 \longrightarrow AgCl + ...$
$200 \times 0.01 \times 1 = 0.1 \times V$
$V = 20$ ml

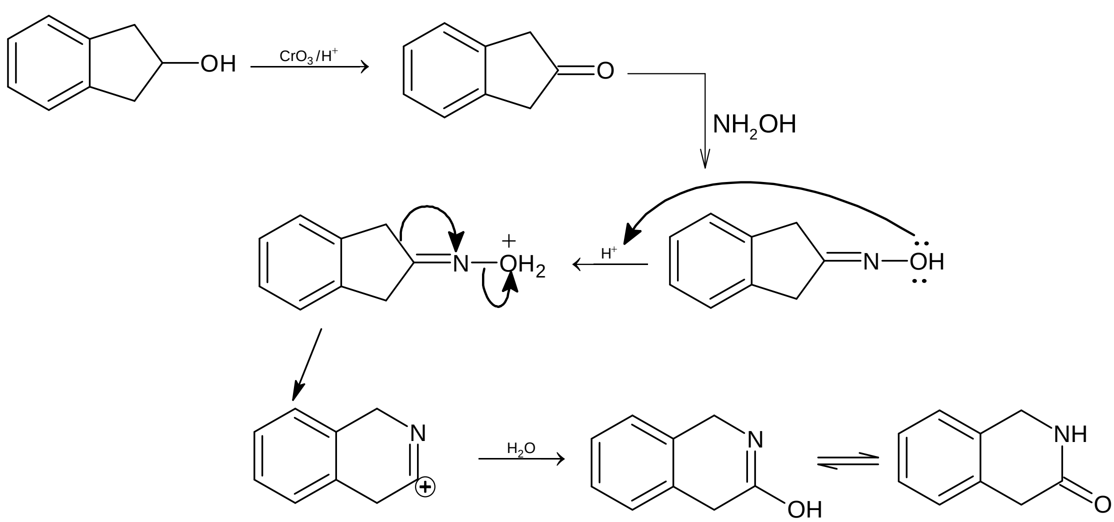
51. C
Sol. $$\log K = \log A - \frac{Ea}{2.303\,RT}$$
$$\tan\theta = \left[-\frac{1}{2.303}\right] = -\frac{Ea}{2.303R}$$
Ea = R = 2 cal/mol.

52. D
Sol. It is $[Ma_2b_2Cd]$ type complex.
6 → geometrical (4 optically inactive and 2 optically active)



53. D

Sol.



54. C

Sol. Aldehydes having one α-H at chiral centre, can undergo racemisation.

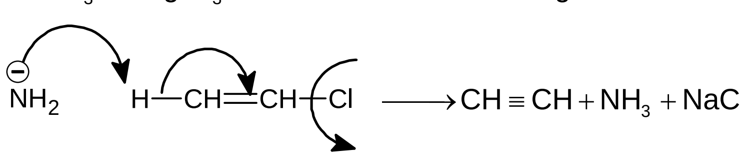
55. A

Sol. Fact

56. A

Sol. $CaC_2 + 2H_2O \longrightarrow C_2H_2 + Ca(OH)_2$

$HC - I_3 + 6Ag + I_3 - CH \longrightarrow CH \equiv CH + 6AgI$



57. D

Sol. $BrF_5 + H_2O \longrightarrow HBrO_3 + HF$

58. A

Sol. $Zn \rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2$

p-orbital have one and d-orbitals have two angular node.

59. C

Sol. $Fe^{+3}(aq) + I^-(aq) \longrightarrow Fe^{2+}(aq) + I_3^-(aq)$

$XeF_6 + SiO_2 \longrightarrow XeOF_4 + SiF_4$

$FeCl_3.6H_2O + 6SOCl_2 \longrightarrow FeCl_3 + 12HCl + 6SO_2$

$2Fe(s) + 3Cl_2(g) \longrightarrow 2FeCl_3$

A, B and D are incorrect.

60. B

Sol. $P_{obs} > P_{ideal}$

$\therefore (BP)_{obs} < (P)_{ideal}$

Mathematics

PART – III

SECTION – A

61. C

Sol. $z = \frac{(2\cos\theta + i)((\sin\theta - 1) + i\cos\theta)}{(\sin\theta - 1)^2 + \cos^2\theta}$

$\therefore$ $2\cos^2\theta + \sin\theta - 1 = 0 \Rightarrow (2\sin\theta + 1)(\sin\theta - 1) = 0$

62. C

Sol. $I_n = \left|x^n e^x\right|_0^1 - n\int_0^1 x^{n-1}e^x dx$

$\Rightarrow I_n = e - nI_{n-1}$

63. A

Sol. $f(g(x)) = x \Rightarrow f'(g(x))\cdot g'(x) = 1$

$\Rightarrow f'(g(0)) = \frac{1}{g'(0)} \Rightarrow f'(1) = \frac{1}{g'(0)}$

64. D

Sol. $(16x)^2 + (4y)^2 + (z)^2 - (16x)(4y) - (4y)(z) - (z)(16x) = 0$

$\Rightarrow \frac{1}{2}\left[(16x - 4y)^2 + (4y - z)^2 + (z - 16x)^2\right] = 0 \Rightarrow 16x = 4y = z$

65. B

Sol. $E = \frac{1}{\sin 20^o} - \frac{1}{\sqrt{3}\cos 20^o} = \frac{\frac{\sqrt{3}}{2}\cos 20^o - \frac{1}{2}\sin 20^o}{\frac{\sqrt{3}}{2}\sin 20^o \cos 20^o} = \frac{\sin(60^o - 20^o)}{\frac{\sqrt{3}}{4}\cdot\sin 40^o}$

66. D

Sol. S = coefficient of x^5 in $(1 + x)^{11}\cdot(1 + x^2)^{11} = {}^{11}C_1\cdot{}^{11}C_2 + {}^{11}C_3\cdot{}^{11}C_1 + {}^{11}C_5\cdot{}^{11}C_0$

67. A

Sol. $(x^2 + 3x + 2)\cdot(x^2 - 9x + 20) = (x^2 - 3x - 4)(x^2 - 3x - 10)$

Let $x^2 - 3x = t$

$\therefore$ $(t + 1)(t - 4)(t - 10) = -30$

By inspection and trial $t = 5$

Hence, other 2 roots are $t = 4 \pm \sqrt{30}$

68. B

Sol. $\pi\cos\theta = n\pi + \frac{\pi}{2} - \pi\sin\theta \Rightarrow \sin\theta + \cos\theta = \frac{2n+1}{2} \Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2n+1}{2\sqrt{2}}$

$\Rightarrow \left|\frac{2n+1}{2\sqrt{2}}\right| \le 1 \Rightarrow n = -1, 0$

$\therefore$ $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

69. A

Sol. $\left\{\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{b}-\left[\vec{a}\ \vec{b}\ \vec{b}\right]\vec{c}\right\}\times(\vec{c}\times\vec{a})=0$

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]\left((\vec{a}\cdot\vec{b})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}\right)=0$

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$ ($\because$ $\vec{a}$ and $\vec{c}$ are not collinear)

70. C

Sol. $m_1=-\frac{2}{3}$; Let one of the lines by y = mx

$\therefore \tan\left(\frac{\pi}{4}\right)=1=\left|\frac{m+\frac{2}{3}}{1-\frac{2m}{3}}\right|=\left|\frac{3m+2}{3-2m}\right|$

$\Rightarrow (3-2m)^2=(3m+2)^2$

$\Rightarrow 9-12m+4m^2=9m^2+12m+4$

$\Rightarrow 5m^2+24m-5=0$

$\Rightarrow \frac{5y^2}{x^2}+\frac{24y}{x}-5=0$

$\Rightarrow -5x^2+24xy+5y^2=0$

$\Rightarrow 5x^2-24xy-5y^2=0$

71. B

Sol. $(x-8)^2+(y+15)^2=(3)^2 \Rightarrow x=8+3\cos\theta,\ y=-15+3\sin\theta$

72. D

Sol. $S=\lim\limits_{m\to\infty}\sum\limits_{n=1}^{m}\tan^{-1}\left(\frac{9}{9n^2+3n+7}\right)=\lim\limits_{m\to\infty}\sum\limits_{n=1}^{m}\tan^{-1}\left(\frac{1}{1+n^2+\frac{n}{3}-\frac{2}{9}}\right)$

$=\lim\limits_{m\to\infty}\sum\limits_{n=1}^{m}\tan^{-1}\left(\frac{\left(n+\frac{2}{3}\right)-\left(n-\frac{1}{3}\right)}{1+\left(n+\frac{2}{3}\right)\left(n-\frac{1}{3}\right)}\right)=\lim\limits_{m\to\infty}\sum\limits_{n=1}^{m}\left[\tan^{-1}\left(n+\frac{2}{3}\right)-\tan^{-1}\left(n-\frac{1}{3}\right)\right]$

73. A

Sol. $120=\frac{1}{2}\cdot10\cdot24\sin A \Rightarrow A=\frac{\pi}{2} \Rightarrow a=26 \Rightarrow s=30 \Rightarrow r=4$

Now, $d=r\ \text{cosec}\left(\frac{A}{2}\right)$

74. C

Sol. $R_1\to\frac{R_1}{x},\ R_2\to\frac{R_2}{y},\ R_3\to\frac{R_3}{z}$

$\therefore\ \Delta=xyz\begin{vmatrix}\frac{x^3+1}{x} & y^2 & z^2\\ x^2 & \frac{y^3+1}{y} & z^2\\ x^2 & y^2 & \frac{z^3+1}{z}\end{vmatrix}$, $C_1\to xC_1,\ C_2\to yC_2,\ C_3\to zC_3$

$\therefore\ \Delta = \begin{vmatrix} x^3+1 & y^3 & z^3 \\ x^3 & y^3+1 & z^3 \\ x^3 & y^3 & z^3+1 \end{vmatrix}$, $C_1 \to C_1 + C_2 + C_3$ and then $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ gives

$\Delta = x^3 + y^3 + z^3 + 1 = 37$

$\therefore$ Permutations of (1, 2, 3)

75. C

Sol. Centre of circle is $C\left(r+\frac{5}{4}, 0\right)$. Equation of circle is $\left(x-\left(r+\frac{5}{4}\right)\right)^2 + y^2 = r^2$

Solving with $y^2 = 5x$, we get $\left(x-\left(r+\frac{5}{4}\right)\right)^2 + 5x = r^2$

$\Rightarrow\ x^2 + \left(\frac{5}{2} - 2r\right)x + \left(\frac{5r}{2} + \frac{25}{16}\right) = 0$, D = 0 $\Rightarrow \left(2r - \frac{5}{2}\right)^2 = 4\left(\frac{5r}{2} + \frac{25}{16}\right)$

76. D

Sol. Sum of digits = 36. Thus number is divisible by 9. For it to be divisible by 36, it should be divisible by 4.

Hence last 2 digits can be 12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76 or 84

Thus, 14 possibilities $\therefore$ Favourable ways = $14 \times 6!$ $\therefore\ P = \frac{14 \times 6!}{8!}$

77. B

Sol. $T:\ y = mx \pm \sqrt{9m^2+1}\ \ \therefore\ 2 = \frac{\sqrt{9m^2+1}}{\sqrt{1+m^2}} \Rightarrow m = -\frac{\sqrt{3}}{\sqrt{5}}$

$e = \frac{2\sqrt{2}}{3}\ \ \therefore\ F\left(2\sqrt{2}, 0\right)$, $RS: y = -\frac{\sqrt{3}}{\sqrt{5}}\left(x - 2\sqrt{2}\right) \Rightarrow \sqrt{5}y + \sqrt{3}x = 2\sqrt{6}$

Let T be foot of perpendicular from O(0, 0) to RS

Then, $OT = \frac{2\sqrt{6}}{2\sqrt{2}} = \sqrt{3}\ \ \therefore\ R'S' = 2\sqrt{4-3}$

78. C

Sol. $N = {}^{9}C_2 \cdot {}^{10}C_2$

79. A

Sol. $PQ: \frac{x}{3}\cos\left(\frac{\alpha-\beta}{2}\right) - \frac{y}{2}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$

$\Rightarrow\ \frac{x}{6}\sec\left(\frac{\alpha+\beta}{2}\right) - \frac{y}{2}\tan\left(\frac{\alpha+\beta}{2}\right) = 1$ compare with $T: \frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

$\therefore$ a = 6, b = 2

80. D

Sol. $|A| = 3^4$

$|\text{adj}(\text{adj } A)| = ||A|^2 \cdot A| = |A|^8 \cdot |A| = |A|^9$

81. B

Sol. Let $\pi_1 : x - 2y + 2z - 3 = 0$, $\pi_2 : x + y + z + 1 = 0$ (Mirror)

Let $\pi_3 : (x - 2y + 2z - 3) + \lambda(x + y + z + 1) = 0$

$\Rightarrow (\lambda + 1)x + (\lambda - 2)y + (\lambda + 2)z + (\lambda - 3) = 0$

Now, π_2 makes same angle with π_1 and π_3

$$\Rightarrow \cos\theta = \frac{1}{3\sqrt{3}} = \frac{3\lambda+1}{\sqrt{3}\sqrt{(\lambda+1)^2+(\lambda-2)^2+(\lambda+2)^2}} \Rightarrow \lambda = -\frac{2}{3}$$

82. A

Sol. E = (p + q)′ + (p′.q) ⇒ p′.q ′ + p′q = p′.(q′ + q) = p′

83. C

Sol. 9 + a + b = 20 ⇒ a + b = 11 ..... (1)

Also, $\frac{26}{5} = \frac{41+a^2+b^2}{5} - 16 \Rightarrow a^2 + b^2 = 65$ ..... (2)

84. D

Sol. x + 300 = h cot α ..... (1)

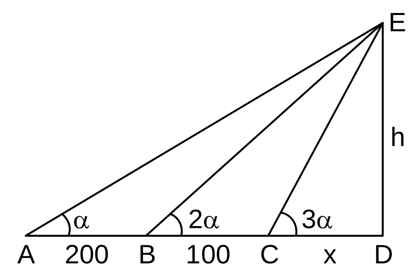
x + 100 = h cot 2α ..... (2)

x = h cot 3α ..... (3)

(1) – (3) ⇒ 300 = h (cot α – cot 3α) ..... (4)

(2) – (3) ⇒ 100 = h (cot 2α – cot 3α) ..... (5)

$$(4) \div (5) \Rightarrow 3 = \frac{\left(\frac{\cos\alpha}{\sin\alpha} - \frac{\cos 3\alpha}{\sin 3\alpha}\right)}{\left(\frac{\cos 2\alpha}{\sin 2\alpha} - \frac{\cos 3\alpha}{\sin 3\alpha}\right)} = \frac{\sin^2(2\alpha)}{\sin^2\alpha}$$



$\Rightarrow \alpha = \frac{\pi}{6}$ now put in equation (4)

85. B

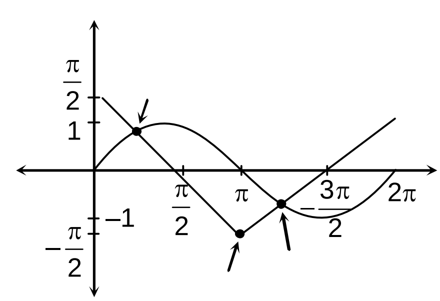
Sol. $$L = \left(\lim_{x\to 0}\left(\frac{(1+4x+x^2)^{\frac{1}{x}} - e^4}{x}\right)\right) - \left(\lim_{x\to 0}\left(\frac{(1+4x-5x^2)^{\frac{1}{x}} - e^4}{x}\right)\right) = L_1 - L_2$$

$$L_1 = \lim_{x\to 0}\frac{e^{\frac{\ln(1+4x+x^2)}{x}} - e^4}{x} = \lim_{x\to 0}\frac{e^{\frac{1}{x}\left((4x+x^2) - \frac{(4x+x^2)^2}{2} + ...\right)} - e^4}{x}$$

$$= \lim_{x\to 0}\frac{e^4\left(e^{\left(x - \frac{x}{2}(4+x)^2 + ...\right)} - 1\right)}{x} = \lim_{x\to 0} e^4\left(\frac{x - \frac{x}{2}(4+x)^2}{x}\right) = -7e^4 \text{ similarly } L_2 = -13e^4$$

86. D

Sol. Shown in the figure



87. A

Sol. $f'(x) < 0$

$\therefore f(x^3 + f(x)) \le f(-f(x) - x^3)$

$\Rightarrow x^3 + f(x) \ge -f(x) - x^3$

$\Rightarrow f(x) + x^3 \ge 0$

$\Rightarrow 3x^2 + 6x - 1 \le 0$

As $x \in Z$, $x \in \{-2, -1, 0\}$

88. B

Sol. $I = \int \cos(8x + x)\cdot\cos^7 x dx = \int(\cos(8x)\cdot\cos x - \sin(8x)\cdot\sin x)\cdot\cos^7 x dx$

$= \int\cos(8x)\cdot\cos^8 x dx - \int\underbrace{\sin(8x)}_{u}\cdot\underbrace{\sin x\cdot\cos^7 x}_{v} dx$

$= \int\cos(8x)\cdot\cos^8 x dx - \left[-\frac{\sin(8x)\cdot\cos^8 x}{8} + \frac{8}{8}\int\cos(8x)\cos^8 x dx\right] = \frac{\sin(8x)\cdot\cos^8 x}{8} + c$

89. C

Sol. $\frac{f(x)}{1+x^2} = 1 + \int_0^x \frac{(f(t))^2}{1+t^2}dt$ differentiating w.r.t. x

$\frac{(1+x^2)f'(x) - 2xf(x)}{(1+x^2)^2} = \frac{f^2(x)}{(1+x^2)} \Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)\frac{1}{y} = 1$

Put $-\frac{1}{y} = t$

$\therefore \frac{dt}{dx} + \left(\frac{2x}{1+x^2}\right)t = 1$, i.f. = $1 + x^2$

$\therefore$ Solution is $-\frac{(1+x^2)}{y} = x + \frac{x^3}{3} + c$ now f(0) = 1

$\therefore c = -1 \Rightarrow y = \frac{-3(1+x^2)}{x^3 + 3x - 3} = f(x)$

90. A

Sol. $S = \int_0^{\frac{t}{2}}(e^{t-x} - e^x)dx = \left|e^{t-x} + e^x\right|_{\frac{t}{2}}^{0} = \left(e^{\frac{t}{2}} - 1\right)^2$

$\therefore \lim_{t\to 0}\frac{S}{t^2} = \lim_{t\to 0}\frac{1}{4}\left(\frac{e^{\frac{t}{2}} - 1}{\frac{t}{2}}\right)^2 = \frac{1}{4}$