

FIITJEE

ALL INDIA TEST SERIES

FULL TEST – V

JEE (Advanced)-2019

PAPER – 2

TEST DATE: 27-01-2019

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A, B, C, D

Sol. $l_1 + l_2 + l_3 + l_4 = 100$ cm

Let n_1, n_2, n_3 and n_4 are fundamental frequencies of these segments respectively.

$$\frac{n_1}{n_2} = \frac{1}{2}, \frac{n_2}{n_3} = \frac{2}{3}, \frac{n_3}{n_4} = \frac{3}{4}, \frac{n_2}{n_4} = \frac{2}{4} = \frac{1}{2}$$

$$n = \frac{1}{2l} \sqrt{\frac{T}{\mu}} \Rightarrow nl = \text{constant}$$

$$n_1 l_1 = n_2 l_2 = n_3 l_3 = n_4 l_4$$

$$l_1 = \frac{n_2}{n_1} l_2, l_3 = \frac{n_2}{n_3} l_2, l_4 = \frac{n_2}{n_4} l_2$$

$$l_1 + l_2 + l_3 + l_4 = 100 \text{ cm}$$

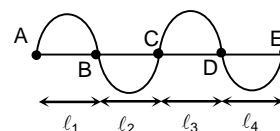
$$2l_2 + l_2 + \frac{2}{3}l_2 + \frac{l_2}{2} = 100$$

$$l_2 = 24 \text{ cm}$$

$$\text{So, } l_1 = 48 \text{ cm}$$

$$l_3 = 16 \text{ cm}$$

$$l_4 = 12 \text{ cm}$$



2. A, B, C, D

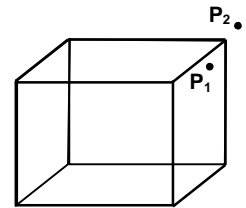
Sol. Using symmetry if charged particle lies at P_1 then $\phi_{OCDE} = \frac{q}{24\epsilon_0}$

$$\text{and } \phi_{OABC} = \frac{\left(\frac{q}{\epsilon_0} - \frac{q}{8\epsilon_0}\right)}{3} = \frac{7q}{24\epsilon_0}$$

If the charge particle lies at P_2 then

$$\phi_{OCDE} = \frac{q}{24\epsilon_0}$$

$$\text{and } \phi_{OABC} = \frac{q}{24\epsilon_0}$$



3. A, B, C

Sol. $AB \rightarrow U\rho = \text{constant}$

$$nC_v T \frac{PM}{RT} = \text{constant}$$

$P = \text{constant}$

Isobaric process

$BC \rightarrow \rho = \text{constant}$

$$\frac{PM}{RT} = \text{constant}$$

$P \propto T$

Isochoric process

$CA \rightarrow U = \text{constant}$

$U = nC_v T = \text{constant}$

$T = \text{constant}$

Isothermal process

4. A, C, D

Sol. Just after cutting the string B, block loses the contact with the rod. taking torque about P

$$mg \frac{\ell}{4} = \left(\frac{m\ell^2}{12} + \frac{m\ell^2}{16}\right)\alpha$$

$$\alpha = \frac{12g}{7\ell}, a_{cm} = \frac{\ell}{4}\alpha = \frac{3}{7}g$$

$$mg - T = ma_{cm}$$

$$T = \frac{4}{7}mg$$

5. A, D

Sol. Work performed = $\int_R^{3R} \frac{1}{2}\epsilon_0 E^2 dV + \int_{5R}^{7R} \frac{1}{2}\epsilon_0 E^2 dV = \frac{kq^2}{3R} + \frac{kq^2}{35R}$

6. A, D

Sol. In both case (A) and (D) the source and observer are relatively at rest. Thus there is no change in the frequency.

7. B
Sol. If first 8m, then 4m, then 2m fired, then velocity of tank

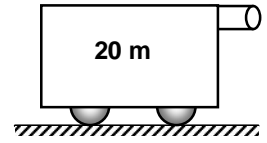
$$v = \left(\frac{8}{20} + \frac{4}{12} + \frac{2}{8} \right) v_0 = \frac{59}{60} v_0$$

If first 8m, then 2m, then 4m fired, then velocity of tank

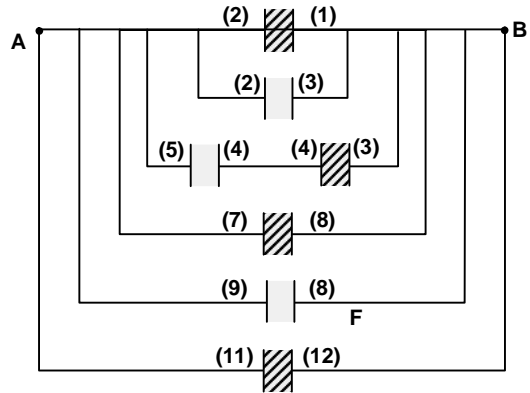
$$v = \left(\frac{8}{20} + \frac{2}{12} + \frac{4}{10} \right) v_0 = \frac{29}{30} v_0$$

If only shell 4m is fired first velocity of shell = $v_0 - \frac{4v_0}{20} = \frac{4}{5} v_0$

If only shell 2m is fired first velocity of shell = $v_0 - \frac{2v_0}{20} = \frac{9v_0}{10}$



8. D
Sol. Apply Kirchoff's Law on the equivalent circuit shown in the diagram.



9. C
Sol. In each case

$$T = \frac{F}{\ell} x$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{F x}{\ell m / \ell}} = \sqrt{\frac{Fx}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{Fx}{m}}$$

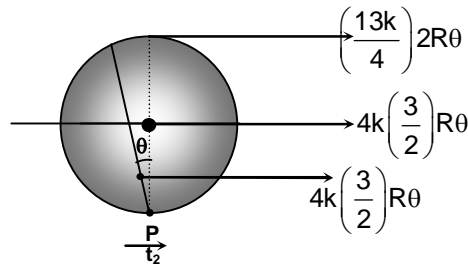
$$\int_0^{\ell} \frac{dx}{\sqrt{x}} = \sqrt{\frac{F}{m}} \int_0^t dt, \quad t = 2\sqrt{\frac{m\ell}{F}}$$

10. C
Sol. (P) Case (i)
Torque about 'P'

$$\frac{3}{2} m R^2 \alpha = -18kR^2 \theta$$

$$\alpha = -\left(\frac{36k}{3m} \right) \theta$$

$$t_1 = \frac{\pi}{6} \sqrt{\frac{3m}{k}}$$



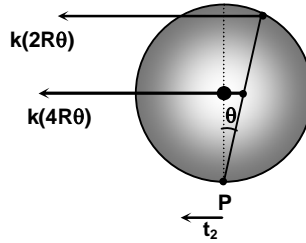
Case (ii)

$$\frac{3}{2}mR^2\alpha = -8kR^2\theta$$

$$\alpha = -\left(\frac{16k}{3m}\right)\theta$$

$$t_2 = \frac{\pi}{4}\sqrt{\frac{3m}{k}}$$

$$T = t_1 + t_2 = \frac{5\pi}{12}\sqrt{\frac{3m}{k}}$$



(Q) $K_{eq1} = 2(8k \cos^2 30^\circ) + 2(8k \cos^2 60^\circ) = 12k + 4k = 16k$

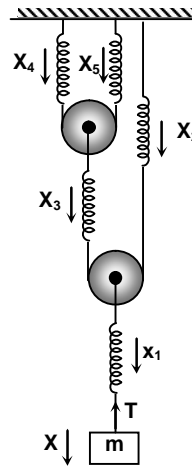
So, $K_{eq} = \frac{(16 \cdot 16)}{32}k = 8k$

(R) $x = x_1 + \frac{x_2 + x_3}{2} + \frac{x_4 + x_5}{4}$

$$\frac{T}{k_{eq}} = \frac{T}{k} + \frac{T}{k} + \frac{2T}{k} + \frac{T}{k} + \frac{2T}{k}$$

$$k_{eq} = \frac{k}{7}$$

$$T = 2\pi\sqrt{\frac{7m}{k}}$$



(S) Block is displaced by 'x' along x-axis.

Torque about P

$$8k(2x_1)\ell = 4k(x - x_1)\frac{\ell}{2}$$

$$x_1 = \frac{x}{9}$$

Torque about Q

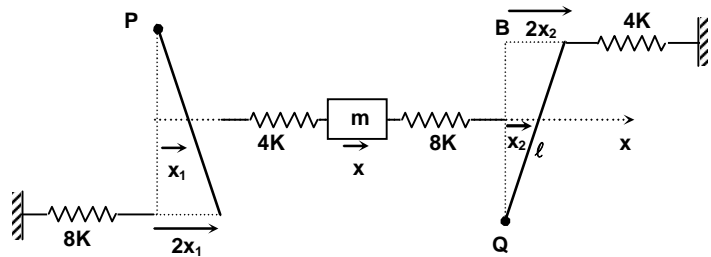
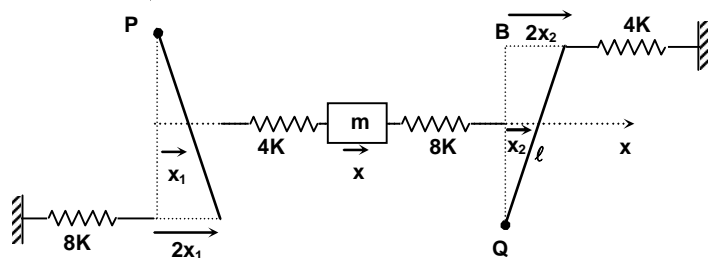
$$4k \cdot 2x_2 \cdot \ell = 8k(x - x_2)\frac{\ell}{2}$$

$$x_2 = \frac{x}{3}$$

$$ma = -\frac{80}{9}kx$$

$$a = -\left(\frac{80k}{9m}\right)x$$

$$T = 2\pi\sqrt{\frac{9m}{80k}} = \frac{3\pi}{2}\sqrt{\frac{m}{5k}}$$



SECTION – D

11. 00000.25

Sol. $T = \frac{2u \sin \theta}{g} = 4 \text{ sec}$

When the ball collide with the wall B, time elapsed

$$= \frac{10}{20} + \frac{15}{10} = 2 \text{ sec}$$

So remaining time = 2 sec

Velocity of the ball after the collision with ball = $\frac{5}{2} = 2.5 \text{ m/s}$

$$e = \frac{2.5}{10} = 0.25$$

12. 00000.50

Sol. $\Delta l_1 + \Delta l_2 = \Delta l$

$$l_1 \alpha_1 \Delta T + l_2 \alpha_2 \Delta T = (l_1 + l_2) \alpha \Delta T$$

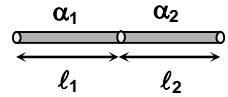
$$\frac{l_1}{l_1 + l_2} = \frac{\alpha - \left(\frac{l_2}{l_1 + l_2} \right) \alpha_2}{\alpha_1} \quad \dots(i)$$

Let $\frac{l_1}{l_1 + l_2} = x$, so $\frac{l_2}{l_1 + l_2} = 1 - x$... (ii)

$$x = \frac{\alpha - (1-x)\alpha_2}{\alpha_1}$$

$$\alpha_1 x = \alpha - \alpha_2 + \alpha_2 x$$

$$x = \left(\frac{\alpha - \alpha_2}{\alpha_1 - \alpha_2} \right) = \frac{17 \times 10^{-6} - 11 \times 10^{-6}}{23 \times 10^{-6} - 11 \times 10^{-6}} = \frac{6}{12} = 0.50$$



13. 00143.75

Sol. When S_1 is closed and S_2 is open

Current through R_1

$$I_1 = \frac{\varepsilon_0}{1 + 3.68} = \frac{\varepsilon_0}{4.68}$$

Potential difference across $\frac{\ell}{2}$ length = $\left(\frac{\varepsilon_0}{4.68} \right) \times 1.84 = \varepsilon$

Similarly in the second case

$$I_2 = \frac{\varepsilon_0}{R_2 + 1} = \frac{468\varepsilon}{184(R_2 + 1)}$$

Potential difference across $\frac{2\ell}{3}$ length of $R_2 = \frac{2}{3} R_2 I_2 = \frac{2}{3} \times \frac{468\varepsilon R_2}{184(R_2 + 1)}$

$$\frac{2}{3} \times \frac{468\varepsilon R_2}{184(R_2 + 1)} = \varepsilon$$

$$64 R_2 = 92$$

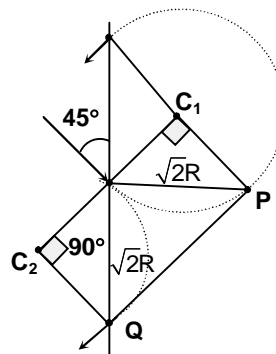
$$\Rightarrow R_2 = 1.4375 \Omega$$

14. 00006.28

Sol. Negatively charged particle exit from magnetic field first and at the time separation is maximum

$$t_Q = \left(\frac{2\pi m}{qB} \right) \frac{\pi/2}{2\pi} = \frac{\pi m}{2qB}$$

 Angle rotated by positively charged particle in the same time = $\pi/2$

 So, separation between A and B = $2R$
 $= 2 \times 3.14 = 6.28$


15. 00002.25

Sol. On releasing the particles, if their acceleration is same, they will stay at a constant distance

Acceleration of charged particle

$$a = \frac{F - 2qE}{2m} \quad \text{and} \quad a = \frac{qE - F}{m}$$

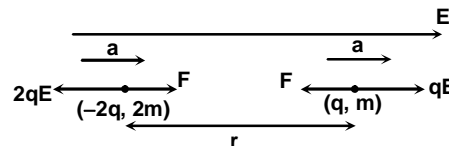
$$\text{So, } F - 2qE = 2qE - 2F$$

$$3F = 4qE$$

$$F = \frac{4qE}{3}$$

When they stay at a constant distance from each other

$$\frac{k \cdot q \cdot 2q}{r^2} = \frac{4qE}{3} \Rightarrow r = \sqrt{\frac{3kq}{2E}}$$



16. 00367.80

Sol. Block on the wedge remains in rest

$$a_{60} = a_{30} = \frac{600 - 300}{90} = \frac{10}{3} \text{ m/s}^2$$

So tension in the rope which is connected with 86.44 kg

$$T = 800 \text{ N}$$

$$f = 800 - 864.4 \sin 30^\circ = 367.80$$

17. 00025.50

 Sol. $6V = 15S$

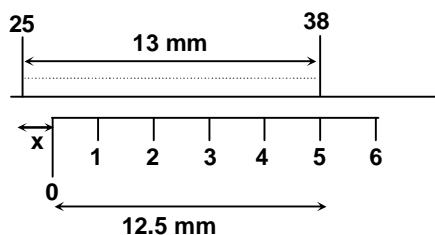
$$V = 2.5 \text{ mm}$$

$$\text{So, } 5V = 12.5 \text{ mm}$$

$$X = 13 \text{ mm} - 12.5 \text{ mm}$$

$$= 0.50 \text{ mm}$$

So, measured value = 25.50 mm



18. 00000.60

Sol. If at O there is dark fringe, then

$$\Delta x = SO''O - SO'O = 2\sqrt{D^2 + d^2} - 2D = \frac{d^2}{D}$$

$$\frac{d^2}{D} = \frac{\lambda}{2} \text{ for } d_{\min}.$$

$$d_{\min} = \sqrt{\frac{D\lambda}{2}} \quad \dots(i)$$

The bright fringe is formed at P. So,

$$\Delta x = SO'P - SO''P$$

$$= D + \sqrt{D^2 + x^2} - \sqrt{D^2 + d^2} - \sqrt{D^2 + (x-d)^2} = \lambda$$

$$\Rightarrow x = \frac{D\lambda}{d}$$

x for d_{\min}

$$x = \frac{D\lambda}{d_{\min}} = \frac{D\lambda}{\sqrt{\frac{D\lambda}{2}}} = \sqrt{2D\lambda} = 0.6 \times 10^{-3}.$$

33. 00001.25

Sol. Number of atoms of X in FCC packing (at corners and face centres of cubic unit cell)

$$= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$

Number of atoms of Y at octahedral voids = 4

Along one body diagonal there are two X atoms and one Y-atom.

$$\text{No. of effective atoms of X after removal} = 4 - 2 \times \frac{1}{8} = \frac{15}{4}$$

$$\text{No. of atoms of Y after removal} = 4 - 1 = 3$$

$$x : y = \frac{15}{4} : 3$$

$$= 5 : 4$$

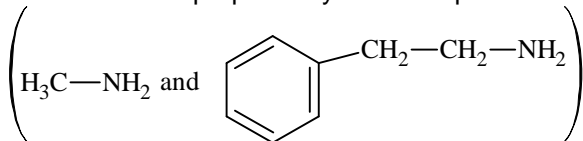
Simplest formula = X_5Y_4

$$a = 5, b = 4, a/b = 1.25$$

34. 00002.50

Sol. Stronger base than benzyl amine (a) = 5.

Amines can be prepared by Gabriel's phthalimide method (b) = 2.



35. 00002.50

36. 00004.98

Sol. Concentration per litre = $\frac{6}{60} + \frac{18}{180} = 0.2$

$$\pi = 0.2 \times 0.083 \times 300$$

$$= 4.98 \text{ bar}$$

Mathematics

PART – III

SECTION – A

37. A, B, C

Sol. Using integration by part

38. A, B, C

Sol.
$$\prod_{r=1}^n \frac{2r-1}{2r} \leq \frac{1}{\sqrt{3n+1}}$$

39. A, B

Sol. $|z| = 1$

40. A, C, D

Sol. Such n digit number will be sum of coefficient of x^{3k} in $(x^2 + x^3 + x^7 + x^9)^n = \frac{4^n + 2}{3}$

41. B, C

Sol.
$$P(x) = \int_0^x f^3(t) dt - \int_0^x f^2(t) dt$$

$$P'(x) = f^3(x) - f(x)^2 = f^2(x) \cdot (f(x) - 1) < 0$$

42. A, C

Sol. B(4, 0) is focus and x = 0 is directrix of parabola

43. D

Sol. $|2A^2B^{-3}| = 8 \times 1^2 \times \frac{1}{27}$

(P) $|A_{4 \times 3} B_{3 \times 4}|$ is 0

(Q) $|A| = 0$ and $|B| = 0$

(R) $AB^T = B^T A$

44. C

Sol. Equation of circle $3x^2 + 3y^2 + 2x + 4y = 0$ and minimum value of $PA + BP + CP + DP = 15$

45. D

Sol.
$$\frac{(x+y)^2}{18} + \frac{(x-y)^2}{2} = 1$$

46. B

Sol. Volume of tetrahedron = $\frac{1}{2} \times \text{base area} \times \text{height}$

SECTION – D

47. 00004.00

Sol. When x is $\frac{\pi}{4}$ then $\sqrt{2} \cos x + 2 > 3$

48. 00010.00

Sol. Inter change m and n

49. 00000.50

Sol. $\lim_{K \rightarrow \infty} Kd_K = \lim_{K \rightarrow \infty} \frac{K(\sqrt{1+K^2} - K)}{\sqrt{2}}$

50. 00008.00

Sol. $\prod_{k=0}^{n-1} \cos\left(\theta + \frac{k\pi}{n}\right) = \frac{\cos n\theta}{(-1)^{\frac{n-1}{2}} \cdot 2^{n-1}}$ if n odd

51. 00001.00

Sol. $\frac{ab - r_1 r_2}{r_3} = \frac{\Delta}{s}$ and $\angle C = \frac{\pi}{2}$

52. 00050.00

Sol. Put $x = r \cos \theta$ and $y = r \sin \theta$ will give area = $\frac{\pi}{4}$

53. 00005.00

Sol. $\vec{p} \cdot \vec{q} \leq |\vec{p}| |\vec{q}|$

54. 00000.95

Sol. $\cos^3 3x + \cos^3 5x = (\cos 5x + \cos 3x)^3$