FIITJEE

ALL INDIA TEST SERIES

FULL TEST – V

JEE (Advanced)-2019

PAPER – 1

TEST DATE: 27-01-2019

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

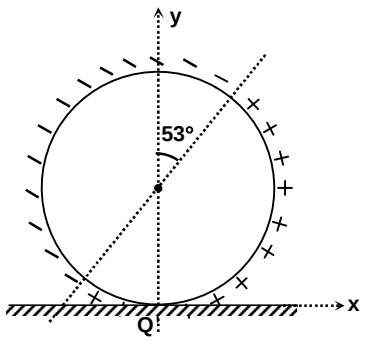
1. A, B

Sol. $\tau_Q = PE\sin 37°$

$= \left(\sigma 2\pi R^2\right)(R)E_0\dfrac{3}{5}$

$= \dfrac{6}{5}\sigma\pi R^3E_0 = \left(\dfrac{5}{3}mR^2\alpha\right)$

$\alpha = \dfrac{18}{25}\dfrac{\sigma\pi RE_0}{m}$

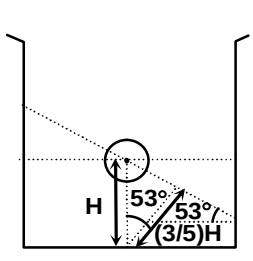


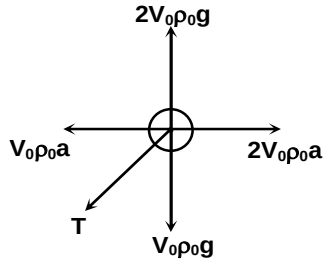
2. A, B, D

Sol. When the vessel starts accelerating then the sphere gets completely immersed in the liquid of density $2\rho_0$.

So,

$T = V_0\left(2\rho_0 - \rho_0\right)\sqrt{a^2 + g^2} = \dfrac{5}{3}V_0\rho_0 g$.





3. A, C, D

Sol. PV = nRT

$$P = \frac{nRT}{V} = \frac{nR(T_0 + V^3)}{V}$$

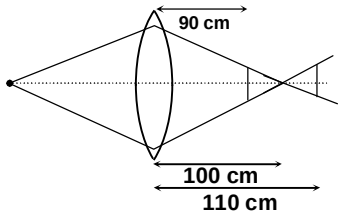
$$\frac{dP}{dV} = 0, \quad V = \left(\frac{T_0}{2}\right)^{1/3}$$

$$P_{min} = \frac{R(T_0 + T_0/2)}{\left(\frac{T_0}{2}\right)^{1/3}} = \frac{3}{2} R T_0^{2/3} 2^{1/3}.$$

4. B, C, D

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

v = 100 cm



5. A, B, C, D

Sol. On increasing frequency stopping potential and maximum kinetic energy of photoelectrons increases.

$K_{max} = hf - \phi = eV_0$

If ϕ is decreased, K_{max} and V_0 both will increases.

With increases in distance between cathode and anode 'f' remains unchanged.

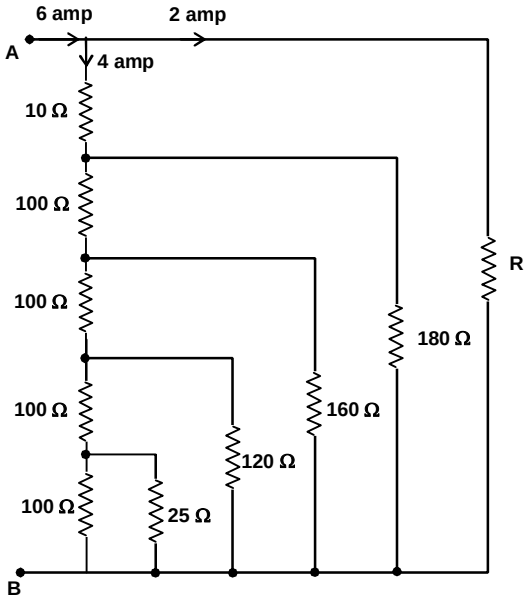
6. A, B, C, D

Sol. Equivalent resistance between P and Q = 100 Ω

So current towards P = $\frac{400}{100} = 4$ ampere

Current through R = 6 – 4 = 2 ampere

So, R = $\frac{400}{2} = 200\ \Omega$



7. A

8. B

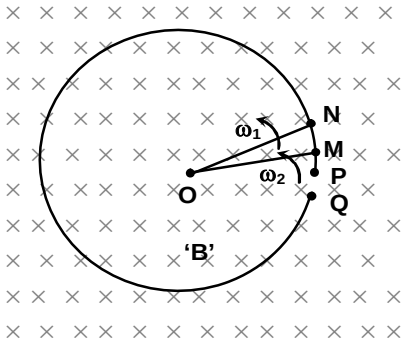
Sol. (for Q. 7 to Q. 8)

By the conservation of angular momentum about 'O'

$I\omega_1 + I\omega_2 = I\omega_0$

Induced emf in the loop

$$\varepsilon = \frac{1}{2}B(\omega_1 - \omega_2)\ell^2$$



Torque on the rod OM about 'O'

$$\tau = \int_0^{\ell}\left[\left(\frac{B\ell^2(\omega_1-\omega_2)}{4R}\right)dr\cdot B\right]r = = \frac{B^2\ell^4}{8R}(\omega_0 - 2\omega_2)$$

$$I\alpha = \frac{B^2\ell^4}{4R}\left(\frac{\omega_0}{2} - \omega_2\right)$$

$$= \int_0^{\omega_2}\frac{d\omega_2}{\left(\frac{\omega_0}{2}-\omega_2\right)} = \frac{3B^2\ell^2}{4mR}\int_0^t dt \Rightarrow \omega_2 = \frac{\omega_0}{2}\left(1 - e^{-\frac{3B^2\ell^2}{4mR}t}\right)$$

Heat produced = $\frac{1}{2}I\omega_0^2 - \frac{1}{4}I\omega_0^2 = \frac{1}{12}m\ell^2\omega_0^2$

9. B

10. A

Sol. (for Q. 9 to Q. 10)

Let P_2 is the initial pressure in upper chamber

$$P_2 = P_0 + \frac{mg}{A} = 2P_0$$

$V_2 = A \times 12 \times 10^{-2}\ m^3$

Let P_2' and V_2' are final pressure and volume of upper chamber and lower piston moved by ℓ cm.

$$V_2' = A(28-\ell)10^{-2}\ m^3$$

$$P_2V_2 = P_2'V_2'$$

$$\Rightarrow P_2' = \left(\frac{24P_0}{28-\ell}\right)$$

Let P_1 is initial pressure in lower chamber

$$P_1 = P_0 + \frac{2mg}{A} = 3P_0$$

$$V_1 = A\times 8\times 10^{-2}\ m^3$$

Let lower piston moved by 'ℓ' cm and P_1' and V_1' are final pressure and volume of lower chamber

$$P_1' = P_2' + \frac{mg}{A} = \frac{24P_0}{28-\ell} + P_0 = \left(\frac{52-\ell}{28-\ell}\right)P_0$$

$$V_1' = A(8+\ell)10^{-2}\ m^3$$

$$P_1V_1 = P_1'V_1'$$

$$3P_0A\times 8\times 10^{-2} = P_0\left(\frac{52-\ell}{28-\ell}\right)A(8+\ell)10^{-2}$$

Solving this ℓ = 4 cm

$$P_1' = \left(\frac{52-4}{28-4}\right)P_0 = 2P_0 = 2\times 10^5\ N/m^2$$

$$P_2' = \frac{24P_0}{28-\ell} = P_0$$

$$\frac{V_2'}{V_1'} = \frac{28-\ell}{8+\ell} = \frac{24}{12} = 2$$

SECTION – D

11. 04645.76

Sol. equation of parabola

$y = kx^2$

$y = h = ka^2$

$k = \left(\frac{h}{a^2}\right)$

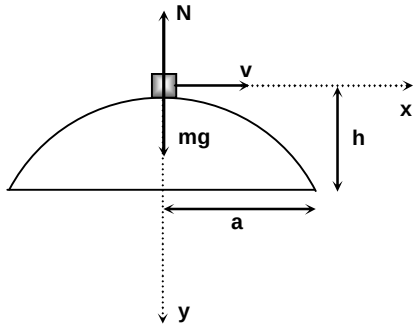
$y = \left(\frac{h}{a^2}\right)x^2$

$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

$R(x=0) = \left(\frac{1}{2k}\right)$

$mg - N = \frac{mv^2}{R}$

$N = mg - \frac{mv^2}{R} = m(g - v^2 \cdot 2k) = 4645.76 \text{ N}$

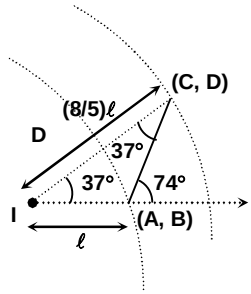


12. 00002.35

Sol. Flux passing through the loop ABCD

$\phi = \int_{\ell}^{(8/5)\ell} \frac{\mu_0}{2\pi}\frac{I}{r}\ell dr = \frac{\mu_0 I\ell}{2\pi}\ell n\left(\frac{8}{5}\right)$

$\phi = \left(\frac{\mu_0\ell}{\pi}\right)\left(\frac{0.47\times10}{2}\right) = 2.35\left(\frac{\mu_0\ell}{\pi}\right)$



13. 00000.16

Sol. When DC source is connected resistance of inductor

$R = \frac{15}{2.5} = 6\Omega$

When AC source connected

$Z = \frac{20}{2} = 10\Omega$

$Z^2 = R^2 + X_L^2$

$X_L = 8\,\Omega$

$2\pi fL = 8$

$L = \frac{8}{2\pi\times25} = \frac{0.16}{\pi}$

14. 00058.40

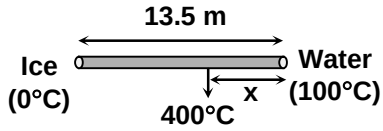
Sol. $E_2 - E_1 = 43.8$

$$\frac{E_1}{(2)^2} - E_1 = 43.8$$

$$E_1 = \frac{-43.8 \times 4}{3} = 58.40\ eV$$

15. 00001.35

Sol. $Q_{ice} = KA\left(\frac{400-0}{13.5-x}\right)t$



So ice melt, $m_{ice} = \frac{Q_{ice}}{L_F} = \frac{KA(400-0)t}{80(13.5-x)}$ …(i)

$$Q_{water} = KA\frac{(400-100)}{x}t$$

$$m_{steam} = \frac{Q_{water}}{L_v} = \frac{KA(400-100)}{540x}t \quad \text{…(ii)}$$

$$m_{ice} = m_{steam}$$

$$\frac{400}{80(13.5-x)} = \frac{300}{540x}$$

$$6(13.5-x) = 54x$$

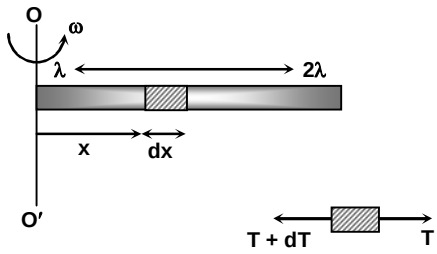
$$6 \times 13.5 = (54x + 6x)$$

$$x = \frac{6 \times 13.5}{60} = 1.35\ m$$

16. 00138.24

Sol. $-\int_0^T dT = \int_{\ell}^{\ell/2}\left[\lambda\left(1+\frac{x}{\ell}\right)dx\right]x\omega^2$

$$T = \frac{2}{3}\lambda\omega^2\ell^2 = \frac{2}{3} \times 4 \times (3.6)^2 \times (2)^2 = 138.24$$



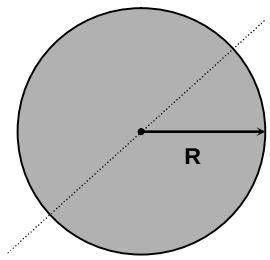
17. 00006.08

Sol. $I_{disc} = I_0 = \frac{mR^2}{4} = kR^4$

$$I_0 = \frac{mR^2}{4}$$

$$I_{PQ} = k\left[(3\sqrt{2})R\right]^4 - \left[I_0 + I_0 + 2\left\{I_0 + m(\sqrt{2}R)^2\right\}\right]$$

$$= 320 - 4mR^2 == 320I_0 - 16I_0 = 304I_0$$

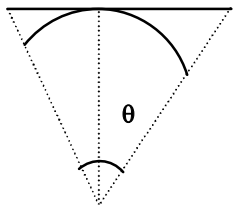


18. 00002.16

Sol. $I = 2\left[\frac{2G\lambda}{R}\sin\left(\frac{\theta}{2}\right)\right]$

Putting all values

I = 2.16 G newton/kilogram



Chemistry

PART – II

SECTION – A
(One or More than one correct type)

This section contains **6** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

19. B

Sol. $2Na_2CrO_4 + H_2O + 2CO_2 \longrightarrow Na_2Cr_2O_7 + 2NaHCO_3$

$3K_2MnO_4 + 2H_2O + 4CO_2 \longrightarrow 2KMnO_4 + MnO_2 + 4KHCO_3$

20. A, B, D

21. A, B, C, D

22. A, B, C, D

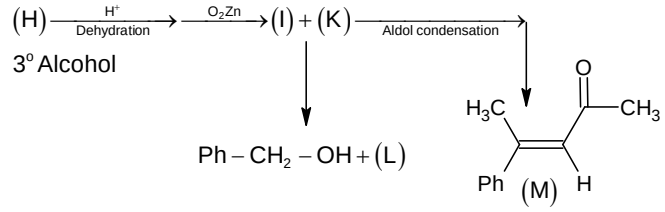
23. B

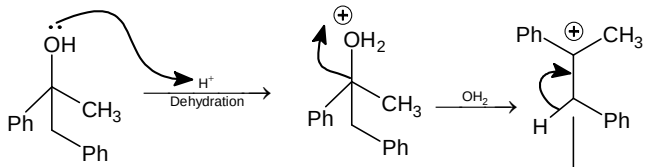
24. C, D

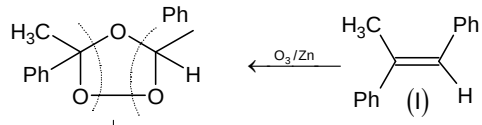
25. B

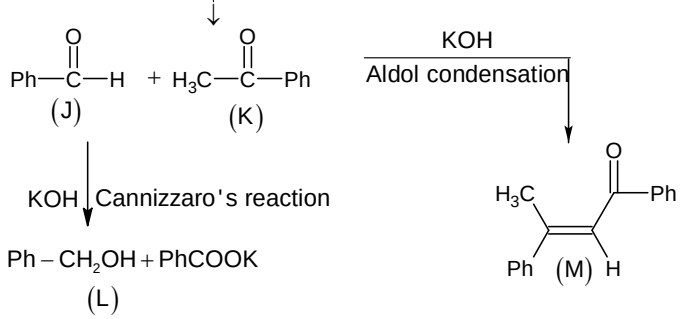
26. A

Sol. (for Q. 25 to Q. 26)









27. B

28. D

Sol. $2AgClO_3 + Cl_2 \xrightarrow{90^oC} 2AgCl + \underset{(x)}{2ClO_2} + O_2$

$2ClO_2 + 2O_3 \longrightarrow \underset{(y)}{Cl_2O_6} + 2O_2$

$Cl_2O_6 + 2NaOH \longrightarrow NaClO_3 + NaClO_4 + H_2O$

$4HClO_4 + P_4O_{10} \xrightarrow{\Delta} 2Cl_2O_7 + 4HPO_3$

SECTION – D

29. 00007.50

Sol. This is a buffer solution where $H_2PO_4^-$ acts as the acid and NaH_2PO_4 is the salt.

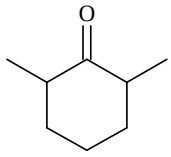
$\therefore pH = pK_2 + \log\frac{\left[HPO_4^-\right]}{\left[H_2PO_4^-\right]}$

$\Rightarrow 7.2 + \log 2$

$\Rightarrow 7.50$

30. 00009.00

Sol. (a) 5

(Except $C_2H_5-\overset{O}{\overset{\|}{C}}-CH_3$, $CH_3-\overset{O}{\overset{\|}{C}}-H$, , 

(b) 4

31. 00003.50

Sol. a = 1; butan-2-ol
b = 3 (ethers)
c = 3 (1^o alcohols)

32. 00001.60

Sol. pH = 12, pOH = 2

$\left[OH^-\right] = 10^{-2}$

Equivalents of $\left[OH^-\right] = 10^{-2} \times 80 \times 10^{-3}$

$= 8 \times 10^{-4}$

$8 \times 10^{-4} = \frac{9.65 \times x}{96500}$

x = 8 sec

Equivalents of oxygen $= \frac{0.28}{5.6} = 0.05$

$0.05 = \frac{965 \times y}{96500}$

y = 5

33. 00002.25

Sol. 6 compounds has 1 mole each
NH_4NO_3 do not give O_2

$$\underset{(1\,\text{mole})}{2LiNO_3} \longrightarrow Li_2O + 2NO_2 + \frac{1}{2}\underset{(0.25\,\text{mole})}{O_2}$$

$$\underset{(1\,\text{mole})}{2Ca(NO_3)_2} \longrightarrow 2CaO + 4NO_2 + \underset{(0.5\,\text{mole})}{O_2}$$

$$\underset{(1\,\text{mole})}{2Pb(NO_3)_2} \longrightarrow 2PbO + 4NO_2 + \underset{(0.5\,\text{mole})}{O_2}$$

$$\underset{(1\,\text{mole})}{2AgNO_3} \longrightarrow 2Ag + 2NO_2 + \underset{(0.5\,\text{mole})}{O_2}$$

$$\underset{(1\,\text{mole})}{2NaNO_3} \longrightarrow Na_2O + \underset{(0.5\,\text{mole})}{O_2} + NO + NO_2$$

Total number of moles of O_2 evolved = 2.25

34. 00001.20

Sol. x = 6; (except H_2CO_3; CH_3CO_2H; H_3BO_3)
y = 5

35. 00035.00

Sol.
$$Co^{2+} + 6NH_3 \rightleftharpoons \left[Co(NH_3)_6\right]^{2+} \qquad K_1 = 10^5$$

$$Co^{3+} + 6NH_3 \rightleftharpoons \left[Co(NH_3)_6\right]^{3+} \qquad K_2 = 10^x$$

$$Co^{3+} + e \longrightarrow Co^{2+} \qquad E^o = 1.9\ V$$

$$\left[Co(NH_3)_6\right]^{2+} \longrightarrow \left[Co(NH_3)_6\right]^{3+} + e \qquad E^o = -0.1\ V$$

$$Co^{3+} + \left[Co(NH_3)_6\right]^{2+} \rightleftharpoons Co^{2+} + \left[Co(NH_3)_6\right]^{3+} \qquad E^o = 1.8\ V$$

$$E^o = \frac{0.06}{1}\log\frac{K_2}{K_1} \text{ (at equilibrium)}$$

$$1.8 = 0.06\log\frac{K_2}{10^5}$$

$$30 = \log\frac{K_2}{10^5}$$

$$K_2 = 10^{35}$$

36. 00900.00

Sol.
$$\log\frac{k_2}{k_1} = \frac{30.8 \times 298 \times 4.606}{2.303 \times 2} \times \left[\frac{308 - 208}{308 \times 298}\right]$$

$$\log\frac{k_2}{k_1} = 1$$

$$\frac{k_2}{k_1} = 10$$

$$k_2 = 10k_1$$

$$\%\ \text{increased (x)} = \frac{10k_1 - k_1}{k_1} \times 100$$

$$= 900.00$$

Mathematics

PART – III

SECTION – A

37. A, B, D

Sol. Let A_n denotes number of codes which ends with 0 or 4, B_n denotes number of codes which ends with 1 or 3 and C_n denotes number of codes which ends with 2

$A_{n+1} = B_n = C_{n+1}$, $B_{n+1} = A_n + 2C_n$, $N = 2^3 \cdot 3^9$

38. B, C

Sol. For same number probability is $\frac{{}^8C_4}{{}^9C_4} \times \frac{1}{{}^8C_4}$

39. A, C

Sol. Let (x, y) be the point on the path followed by dog, then $\frac{dy}{dx} = \frac{y-5t}{x}$

$xy' - y = -5t$ ..... (1)

On differentiating equation (1), w.r.t. x

$xy'' = -5\frac{dt}{dx} = \frac{\sqrt{1+y'^2}}{2}$

40. B

Sol. Use IVT

Let $h(x) = x^{11} - f(x)$

$h(0) < 0$ and $h(1) > 0$

41. A, B

Sol. Locus of the mid-point is $\frac{x^2}{\left(\frac{a^4e^4}{4a^2}\right)} + \frac{y^2}{\left(\frac{a^4e^4}{4b^2}\right)} = 1$

42. A, B

Sol. Let P, Q be (2, 0) and (6, 0) respectively

43. D

Sol. Equation of L is $\frac{x-1}{4} = \frac{y-1}{-3} = \frac{z-1}{-1}$

44. D

Sol. Required line passes through (1, 1, 1) and parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

45. C

Sol. $f(x) = \int_0^{\pi/2} \left(x\sin t + (x\sin t)^3 + ...\right)dt = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

46. D

Sol. $f(x) = \int_0^{\pi/2} \left(x\sin t + (x\sin t)^3 + ...\right)dt = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

SECTION – D

47. 00000.00

Sol. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{(\cos 2\theta)^{2r-1}}{2r-1} = \ell n\cot\theta$

48. 00001.00

Sol. Expanding $(e^x + 1)^n - (e^x - 1)^n$ in two ways

49. 00001.00

Sol. $(ae^{-iB} + be^{iA})^3 = (c + i(b\sin A - a\sin B))^3 = c^3$

50. 00002.00

Sol. Use integration by parts, then use expansion

51. 00006.00

Sol. $\lim\limits_{n\to\infty}\prod\limits_{r=1}^{n}\left(1-\frac{d^2}{a_r^2}\right) = \lim\limits_{n\to\infty}\frac{(8-d)\cdot(8+nd)}{8\cdot(8+(n-1)d)} = \frac{8-d}{8}$

52. 00023.00

Sol. $P(x) = Q(x)(x-1)(x-2) + 2x + 3$

$P(10) = 72n + 23$

53. 00001.00

Sol. From similar ΔAEM and ΔAPE

$\frac{AE}{AP} = \frac{AM}{AE}$

$\Rightarrow AE^2 = AM\cdot AP = (AD - r)AP$ ..... (1)

AB both circles touches each other $(R - r)^2 = r^2 + OD^2 = r^2 + (AD - R)^2$

$AE = AD$

54. 00351.00

Sol. ${}^7C_7 + {}^7C_1[10\times 2\times 2] + {}^7C_4\times 2$