FIITJEE

ALL INDIA INTEGRATED TEST SERIES

HALF COURSE TEST – IV

JEE (Main)-2020

TEST DATE: 31-03-2019

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. B

Sol. The sum is expressed upto minimum number of digits after decimal point, i.e., two digits in this case.

2. A

3. B

4. D

Sol. Distance $= \left(\frac{1}{2}\times 3\times 2\right)+\left(\frac{1}{2}\times 1\times 2\right)+\left(1\times 1\right)=5\text{m}$

5. C

6. C

Sol. At $t=0, y_1 = 4\text{m}$

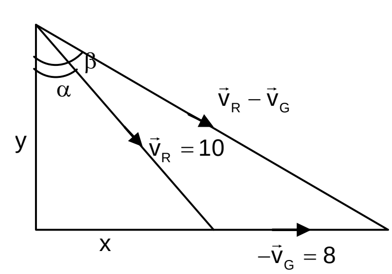
At $t=3, y_2 = 2(3)^2+3(3)+4=31\text{m}$

$\therefore \quad v_{av} = \frac{y_2-y_1}{t_2-t_1}=\frac{31-4}{3-0}=\frac{27}{3}=9\text{ m/s}$

7. B

Sol. $\sin\alpha = \frac{x}{v_R}; x = u_R \sin\alpha$

$\cos\alpha = \frac{y}{v_R}; y = v_R\cos\alpha; \tan\beta = \frac{x+v_G}{y}$



8. D

Sol. It can be observed from Figure that P and Q shall collide if the initial component of velocity 'P' on inclined plane i.e., along incline u = 0 i.e., particle projected perpendicular to incline.

$\therefore$ Time of light, $T = \frac{2u}{g\cos\theta}$

$\therefore$ $u = \frac{gT\cos\theta}{2} = 10\ m/s$

9. A

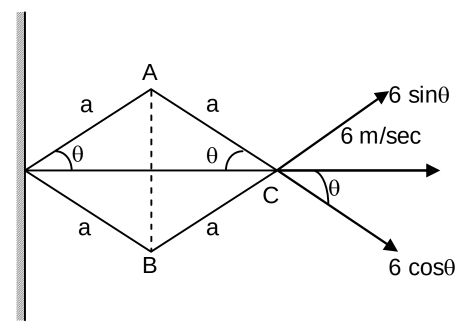
Sol. $\frac{1}{K_{eq}} = \frac{1}{3K} + \frac{1}{4K} = \frac{7}{12K}$; $T = \frac{12K}{7}x_0$

$T = 4Kx_1$; $\frac{12}{7}K/x_0 = 4K/x_1$; $x_1 = \frac{3}{7}x_0$

10. C

Sol. $W = \left(\frac{v_C}{2a\sin\theta}\right)$

$v_A = W \times r_I$



11. B

12. C

Sol. Velocity acquired by the body sliding down the inclined plane is

$v_A = \sqrt{2hg} = \sqrt{2g(4a)} = \sqrt{8ga}$

Using the law of conservation of energy:

$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(a)$

$\frac{1}{2}m(8ga) = \frac{1}{2}mv_B^2 + mga$

$mv_B^2 = 6mga$

$\frac{mv_B^2}{a} = 6mg$

$F_C = 6mg$

But, $F_T = mg$

The resultant force on the body is given by

$F = \sqrt{F_C^2 + F_T^2} = \sqrt{36(mg)^2 + 1(mg)^2} = \sqrt{37}\ mg$

13. A

Sol. $V = x^2 - 3x$

At equilibrium, the P.E. of system is minimum. Therefore,

$\frac{dV}{dx} = 0$

$\frac{d}{dx}\left(x^2 - 3x\right) = 0$

$2x - 3 = 0$

$x = \frac{3}{2} = 1.5m$

14. B

Sol. Maximum frictional force between the slab and the block

$f_{max} = \mu N = \mu mg$

$\frac{1}{4} \times 4 \times 10 = 10\ N$

m = mass of upper block

Evidently, $F < f_{max}$

So, the two bodies will move together as a single unit. If 'a' be their combined acceleration, then

$a = \frac{F}{m+M} = \frac{6}{4+5} = \frac{6}{9} = \frac{2}{3} m/s^2$

Therefore, frictional force acting can be obtained as

$f = Ma = \frac{2}{3} \times 5N = \frac{10}{3} N$

Using $s = \frac{1}{2}at^2$, find s_2 and s_3

$W = \left(s_3 - s_2\right) f$

$s_2 = \frac{1}{2} \times \frac{2}{3}(2)^2 = \frac{4}{3}$

and $s_3 = \frac{1}{2} \times \frac{2}{3} \times (3)^2 = 3$

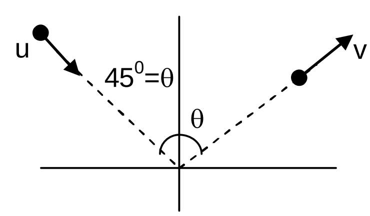
Therefore work done by friction $W = \left(s_3 - s_2\right) f$

$= \frac{10}{3}\left[3 - \frac{4}{3}\right] = \frac{50}{9} = 5.55\ J$

15. A

Sol. $u \sin 45^0 = v \sin \phi$

$v \cos \phi = e(u \sin 45^0)$



16. C

Sol. Given that from graph

$\frac{1}{2} mV_m^2 = 15 \times 10^{-3}$

$V_m = \sqrt{0.150}\ m/s$

$A\omega = \sqrt{0.150}\ m/s$

$L\theta_m . \sqrt{\frac{g}{L}} = \sqrt{0.150}\ m/s$

$$\Rightarrow \sqrt{gL} = \frac{\sqrt{0.150}}{100 \times 10^{-3}}$$

$$\Rightarrow L = \frac{0.150}{0.1} = 1.5 \text{ m}$$

17. B

Sol. If sphere is displaced by x in upward direction from is equilibrium position then increase in weight is λxg due to mass of chain.

Increase in buoyant force due to this is $= \frac{\lambda xg}{7}$

$\Rightarrow$ excess force on chain will be $\left(\lambda xg - \frac{\lambda xg}{7}\right)$ in down ward direction.

If a is acceleration of system then its motion equation is

$$\Rightarrow \frac{6\lambda g}{7} x = (m + \lambda h)a$$

$$a = -\frac{6\lambda g}{7(m + \lambda h)}.x$$

[-ve sing shows restoring tendency]

Comparing this with acceleration of SHM $a = -\omega^2 x$; we get $\omega = \frac{6\lambda g}{7(m + \lambda h)}$

$$\&T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{7(m + \lambda H)}{6\lambda g}}$$

using $h = \frac{7m}{3\lambda}$ at equilibrium

$$T = 2\pi\sqrt{\frac{7.10m}{18\lambda g}} = \frac{2\pi}{3}\sqrt{\frac{35m}{\lambda g}}.$$

18. D

Sol. We use

$$f = \frac{n}{2\ell}\sqrt{\frac{T_1}{\mu}} \qquad ....(i)$$

$$f = \frac{n+1}{2\ell}\sqrt{\frac{T_2}{\mu}} \qquad ....(2)$$

From equation (1) and (2) eliminate 'n' to get μ.

$$\mu = \frac{T_1 T_2}{4f^2\ell^2\left(\sqrt{T_1} - \sqrt{T_2}\right)^2}$$

19. B

Sol. As we know

$$n_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Initially $L = 24$ cm, $T = kx = k(4\text{cm}); \mu = \frac{m}{24}$

$$n_0 = \frac{1}{2 \times 24}\sqrt{\frac{4k}{(m/24)}} = 0.20\sqrt{\frac{k}{m}}$$

When it is stretched to the length 26 cm

$L = 26\text{ cm}$; $T = K(6\text{cm}); \mu = \frac{m}{26}$

$n_0^{'} = \frac{1}{2\times 26}\sqrt{\frac{6k}{(m/26)}} = 0.24\sqrt{\frac{k}{m}}$

$\Rightarrow n_0^{'} > n_0$

20. C

Sol. We use

$\frac{59-5}{95-5} = \frac{T_C - 0}{100}$

$\Rightarrow \frac{54}{90} = \frac{T_C}{100}$

$\Rightarrow T_C = 60^o$

21. A

Sol. From ideal gas equation,

$pV = nRT$

$p = \frac{nRT}{V} = \frac{nR(T_0 + aV^3)}{V}$

$p = \frac{nRT_0}{V} + nRaV^2$

$\frac{dp}{dV} = 0$

$\frac{-nRT_0}{V^2} + 2anRV = 0$

$\frac{T_0}{V^2} = 2aV$

$V^3 = \frac{T_0}{2a}$

$V = \left(\frac{T_0}{2a}\right)^{1/3}$

22. B

Sol. Processes BC and AD are isochoric so at these process density as well as volume remain constant and for AB and CD volume and density inversely varies hence option (B) is correct.

23. A

Sol. From Newton's law of cooling

$\frac{dT}{dt} = -k(T - T_0)$

$\Rightarrow \int_{365}^{361} \frac{dT}{T - T_0} = -k\int_0^{120} dt$

$\Rightarrow \ln\left(\frac{361-293}{365-293}\right) = -k(120-0)$

$k = 4.76\times 10^{-4}$

To cool from 344 K to 342 K

$$\int_{344}^{342} \frac{dT}{T-293} = -k\int_{t_1}^{t_2} dt$$

$$\Rightarrow \ln\left(\frac{342-293}{344-293}\right) = -\left(4.76\times10^{-4}\right)\left(t_2-t_1\right)$$

$$\Rightarrow t_2 - t_1 = 84 \text{ s}$$

24. D

Sol. Because the tuning fork is in resonance with air column in the pipe closed at one end, the frequency is $n = \frac{(2N-1)v}{4\ell}$ where N = 1, 2, 3...... corresponds to different mode of vibration putting n = 340 Hz, v= 340 m/s, the length of air column in the pipe can be

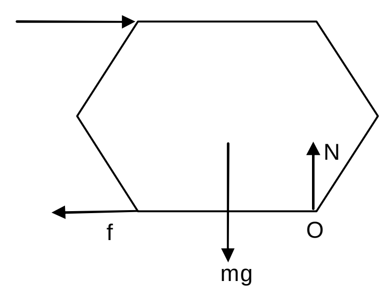
$$\ell = \frac{(2N-1)340}{4\times340} = \frac{(2N-1)}{4}m = \frac{(2N-1)\times100}{4} \text{ cm}$$

For N = 1, 2, 3,…………… we get l = 25 cm, 75 cm, 125 cm……….
As the tube is only 120 cm long, length of air column after water is poured in it may be 25 cm or 75 cm only, 125 cm is not possible, the corresponding length of water column in the tube will be (120 – 25) cm = 95 cm or (120 – 75) cm = 45 cm.
Thus minimum length of water column is 45 cm.

25. A

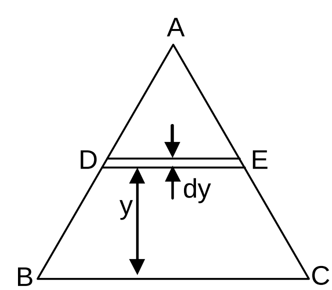
Sol. For toppling $F\times\sqrt{3}\, l \ge mg\times\frac{1}{2}$. The block will not slip if

$F < \mu mg$ Block will topple $\mu > \frac{1}{2\sqrt{3}}$



26. B

Sol. Surface mass density $\sigma = \frac{M}{\left(\frac{1}{2}bh\right)} = \frac{2M}{bh}$



From Δ ADE and Δ ABC

$$\frac{DE}{BC} = \frac{h-y}{n}$$

$$\Rightarrow \quad DE = (h-y)\frac{b}{h}$$

$$dA = (h-y)\left(\frac{b}{h}\right)dy$$

$$dI = \frac{2M}{bh}(h-y)\left(\frac{b}{h}dy\right)y^2$$

$$I = \int_0^h \frac{2M}{h^2}(hy^2 - y^3)dy = \frac{Mh^2}{6}$$

27. B

Sol. $dW = \vec{F}.\overrightarrow{ds} = m\vec{E}.\overrightarrow{ds}$

$= ma(y\,\hat{i} + x\,\hat{j}).(\hat{i}\,dx + \hat{j}\,dy)$

$= ma(y\,dx + x\,dy)$

$= ma\,d(xy)$

$\therefore\; W = ma \int_0^{x_0}\int_0^{y_0} d(xy) = ma\,x_0y_0$

28. C

Sol. Applying gauss theorem for gravitational field

$\int \vec{E}.\overrightarrow{ds} = -4\pi G(m)$

$m =$ enclosed mass

For $r = 2a$ Mass inside Gaussian surface

$\frac{4}{3}\pi\left(\frac{a}{3}\right)^3 d_0 + \frac{4}{3}\pi\left(a^3 - \frac{a^3}{81}\right)\frac{d_0}{2}$

$= \frac{4}{3}\pi a^3\,d_0\left(\frac{43}{81}\right)$

$\Rightarrow\; E \times 4\pi(2a)^2 = 4\pi G\left(\frac{4}{3}\pi a^3\,d_0\right)\left(\frac{43}{81}\right)$

$E = \frac{14}{81}Gd_0\pi a = 0.17\text{ K}$

Where $K = Gd_0\pi a$

Similarly, at $r = \frac{a}{2}$

$E = \frac{35Gd_0\pi a}{81} = 0.43\text{ K}$

At $r = \frac{a}{4}$

$E = \frac{Gd_0\pi a}{3} = 0.33\text{ K}$

Hence option (C) is correct.

29. C

Sol. Length of the rod inside the water $= 1.\sec\theta = \sec\theta$

Up thrust $F = \left(\frac{2}{2}\right)\sec\theta\left(\frac{1}{500}\right)(1000)(10)$

$F = 20\sec\theta,\; W = 20\text{ N}$

For rotational equilibrium of rod net torque about 'O' is zero.

$\therefore\; F\left(\frac{\sec\theta}{2}\right)\sin\theta = W\sin\theta$

$F = 20\sqrt{2}\text{ N}$

For vertical equilibrium of rod, the force exerted by the hinge on the rod $= 20\sqrt{2} - 20 = 8.3\text{ N}$ downwards.

30. C

Sol. The rate of flow of liquid through a capillary tube is:

$$V = \frac{\pi P r^4}{8\eta l}$$

When capillary tubes are connected in series, the rate of flow of liquid through them remains the same or constant. Therefore,

$$P \propto \frac{1}{r^4} \Rightarrow \frac{P_1}{P_3} = \left(\frac{r_3}{r_1}\right)^4$$

$$\frac{P_1}{8.1} = \left(\frac{0.6}{0.3}\right)^4 = 16$$

$P_1 = 16 \times 8.1 = 129.6$ mm of Hg

Chemistry

PART – II

SECTION – A

31. C

Sol. Due to stability of compound by removing H^+ ion.

32. C

Sol. Those compound which are more acidic than phenol gives CO_2 after reaction with $NaHCO_3$.

33. B

34. A

Sol. After bromonation product does not superimpose on its mirror image.

35. D

Sol. Reduction of alkynes give alkanes.

36. B

37. D

Sol. Coupling takes place, hence (D).

38. B

Sol. Circumference of 3rd orbit = $2\pi r_3$

According to Bohr, angular momentum of electron in 3rd orbit is

$$mvr_3 = 3\frac{h}{2\pi}$$

or $\frac{h}{mv} = \frac{2\pi r_3}{3}$

By de Broglie equation,

$$\lambda = \frac{h}{mv}$$

$$\therefore \quad \lambda = \frac{2\pi r_3}{3}$$

$$\therefore \quad 2\pi r_3 = 3\lambda$$

i.e., circumference of 3rd orbit is three times the wavelength of electron or number of waves made by Bohr electron in one complete revolution in 3rd orbit is three.

$\therefore$ (B) is the correct answer.

39. C

Sol. $T = \frac{2\pi r_n}{u_n} = \frac{2\pi r_1 \times n^2}{u_1 / n}\left(\because r_n = n^2 \times r_1\right) or\ T \propto n^3;$

$$\therefore \frac{T_1}{T_2} = \frac{1^3}{n^3} = \frac{1}{8} \ (n = 2)$$

40. B

Sol. Let $T_1 > T_2$; final pressure will be same in both container, but different from the initial pressure which was same in both container. Initial pressure in both container = P_1 given. Let final pressure in both container = P_A. Let x mole transfer from A to B vessel.

$\therefore \quad P_A V = (n-x)\, RT_1$ ………1

and $P_A V = (n+x)\, RT_2$ ………2

By Eq. 1 and 2

$\therefore \quad x = \dfrac{n(T_1 - T_2)}{T_1 + T_2};\; V = \dfrac{nRT_1}{P_1}$

Put the value of x and b in equation 1

$\therefore \quad P_A \times \dfrac{nRT_1}{P_1} = \left(n - \dfrac{n(T_1 - T_2)}{(T_1 + T_2)}\right) RT_1 \quad P_A = \dfrac{2P_1T_2}{T_1 + T_2}$

41. A

Sol. H_2SO_4 is 86% by weight

$\therefore \quad Weight\ of\ H_2SO_4 = 86\ g\ Weight\ of\ solution = 100\ g$

$\therefore \quad Volume\ of\ solution = \dfrac{100}{1.787} mL = \dfrac{100}{1.787 \times 1000} liter$

$\therefore \quad M_{H_2SO_4} = \dfrac{86}{98 \times \dfrac{100}{1.787 \times 1000}} = 15.68$

Let V mL of this H_2SO_4 are used to prepare 1 liter of 0.2 M H_2SO_4

$\therefore \quad mM\ of\ concentrated\ H_2SO_4 = mM\ of\ dilute\ H_2SO_4$

$V \times 15.68 = 1000 \times 0.2$

$\therefore \quad V = 12.75\ mL$

42. A

Sol. Redox change is,

$O_2^{-1} + 2e \rightarrow 2O^{2-}$

$2I^- \rightarrow I_2 + 2e$

$2S_2^{2+} \rightarrow S_4^{+5/2} + 2e$

$I_2 + 2e \rightarrow 2I^-$

Meq. Of $H_2O_2 = Meq.\,of\ I_2 = Meq.\,of\ Na_2S_2O_3 = 20 \times 0.1 = 2$

$\therefore \quad N_{H_2O_2} = \dfrac{Meq.}{Volume\ in\ ml} = \dfrac{2}{25}$

Volume strength (x) = $N \times 5.6$

$\therefore$ Volume strength = $\dfrac{2}{25} \times 5.6 = 0.448$ volume

43. B

Sol. He is a monoatomic gas, so, its $\gamma = 1.67$. For an adiabatic expansion

$P_2V_2^{y} = P_1V_1^{y}$

or $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{y} = 1\times\left(\frac{1}{100}\right)^{1.67}$

$P_2 = 4.57\times10^{-4}$ atmosphere

And $T_2V_2^{y-1} = T_1V_1^{y-1}$

or $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{y-1} = 300\times\left(\frac{1}{100}\right)^{1.67-1}$

$T_2 = 13.7\ K = -259.3^{o}C$

44. D

Sol. Amount of energy unused = $\frac{1560}{2} = 780\ kJ$

Amount of water perspired = $\frac{Energy\ left\ unused}{Enthalpy\ of\ evaporation\ of\ water}$

$= \frac{780\ kJ}{44\ kJ\ mol^{-1}} = \frac{780}{44}\ mole$

$= \frac{780\times18}{44} = 319.14\ g$

45. B

Sol. $N_2O_4(g) \qquad 2NO_2(g)$

Equilibrium $1-\alpha \qquad 2\alpha$

Where α is the degree of dissociation

$\frac{(P_2)\ Pressure\ of\ N_2O_4\ at\ 600\ K}{(P_1)\ Pressure\ of\ N_2O_4\ at\ 300\ K} = \frac{600}{300}$ ($\because$ V is constant)

$\therefore\ P_2 = 2\ atm$

After dissociation of N_2O_4 at 600K,

$P_{N_2O_4} = 2(1-\alpha) = 2-2\alpha$

$P_{N_2O_4} = 2\times2\alpha = 4\alpha$

Total pressure = $2-2\alpha+4\alpha = 2+2\alpha$

$2+2\alpha = 2.4$ (Given)

$\alpha = 0.2$

$\therefore\ Percentage\ dissociation = 20\%$

$\therefore$ (B)

46. C

Sol. $pH = pK_a + \log\frac{[Salt]}{[Acid]}$

$\therefore \quad 4.5 = 4.2 + \log\frac{[Salt]}{[Acid]} = \log\frac{[Salt]}{[Acid]} = 0.3$

(since log 2 = 0.3)

$\therefore \quad \frac{[Salt]}{[Acid]} = 2$

47. D

48. B

Sol. $X \rightarrow X^+; IE_1;$ $X \rightarrow X^-; EA_1$

$IE_1 = \frac{2E_1}{A_0}$ $EA_1 = \frac{2E_2}{A_0}$

49. A

Sol. CO_3^{2-} XeF_4 I_3^- NCl_3 $BeCl_2$

sp^2 sp^3d^2 sp^3d sp^3 sp

50. D

Sol. Si does not form π -bond with O.

51. B

Sol. Size of S > O hence, (S-S) B.L. > $(O-O)$ B.L. Due to $-I$ effect of F, e^- density of $O\downarrow$ and hence, $l.p.\ repulsions \downarrow\ and\ O-O$ bond becomes shorter.

52. C

Sol. Based on E^o values.

53. C

54. C

55. A

56. D

57. D

58. B

59. B

Sol. (A) $HOMO\ \ for\ N_2 \Rightarrow \sigma BMO$

(B) Colour of halogens is due to $HOMO-LUMO$ transition.

60. C

Sol. $C_2H_5OH > CH_3OCH_3$

(B. P. order, due to H-bonding)

Same reason for $NH_3 > PH_3$.

In C & D, Hydration energy > lattice energy

Mathematics PART – III

SECTION – A

61. C

Sol. Any pt on the curve xy = c^2 is $P\left(ct, \frac{c}{t}\right)$. The reflection $Q(x_1, y_1)$ of this point P with respect to the line x + y = 4 is

$$\frac{x_1 - ct}{1} = \frac{y_1 - \frac{c}{t}}{1} = \frac{-2\left(ct + \frac{c}{t} - 4\right)}{2} \quad ....(i)$$

The required curve is the locus of (x_1,y_1) of aften eliminating 't' from equation (i), use have

$x_1 = 4 - \frac{c}{t}$ and $y_1 = 4 - ct$

$\Rightarrow c^2 = (4 - x_1)(4 - y_1)$

$\Rightarrow \boxed{(x-4)(y-4) = c^2}$

62. A

Sol. $\because \quad b = \frac{2ac}{a+c} \Rightarrow \frac{x}{a} + \frac{y(a+c)}{2ac} + \frac{1}{c} = 0$

$\Rightarrow \quad 2cx + ya + cy + 2a = 0$

$\Rightarrow \quad (2x + y) + \frac{a}{c}(y + 2) = 0$

$2x + y = 0 \;\&\; y + 2 = 0$

The fixed point is (1, –2).

63. B

Sol. In an equilateral triangle, the circumcenter and the incentre are the same point.

$\therefore$ incentre $= (-g, -f)$.

Also $1^2 + 1^2 + 2g + 2f + c = 0$ (given)i.e., $c = -2(g + f + 1)$

Also, in an equilateral triangle, circum-radius = 2 × in-radius

$\therefore$ in-radius $= \frac{1}{2} \times \sqrt{g^2 + f^2 - c}$.

$\therefore$ The equation of the incircle

$$(x+g)^2 + (y+f)^2 = \frac{1}{4}(g^2 + f^2 - c)$$

$$\Rightarrow x^2 + y^2 + 2gx + 2fy + 3\frac{g^2}{4} + 3\frac{f^2}{4} + \frac{c}{4} = 0$$

Simplify.

64. B

Sol. $f(x,y) \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

Now, $f(0,\lambda) \equiv \lambda^2 + 2f\lambda + c = 0$, and its roots are 2, 2.

$\therefore \quad 2 + 2 = -2f, 2 \times 2 = c$, i.e., $f = -2, c = 4$

$f(\lambda,0) \equiv \lambda^2 + 2g\lambda + c = 0$, and its roots are $\frac{4}{5}, 5$

$\therefore \quad \frac{4}{5}+5=-2g, \frac{4}{5}\times 5=c.$ Thus $g=-\frac{29}{10}, f=-2$

65. D

Sol. Here, the centre $=\left(\frac{p}{2},\frac{q}{2}\right)$ so, (p, q) and (0, 0) are the ends of a diameter. As the two chords are bisected by the line y = 0, the chords will cut the circle at the points $(x_1,-q)$ and $(x_2,-q)$ where x_1, x_2 are real.

Clearly, the line joining these points is $y=-q$. Solving

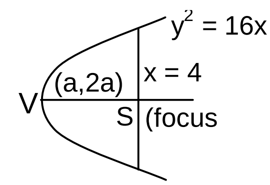
$y=-q$ and $x^2+y^2=px+qy$, we get

$x^2-px+2q^2=0$, whose roots are x_1, x_2.

For real distinct roots, $D>0$, i.e., $p^2-8q^2>0$.

66. B

Sol. (a, 2a) is an interior point of $y^2-16x=0$ if $(2a)^2-16a<0$, i.e., $a^2-4a<0$. V (0,) and (a, 2a) are on the same side of x – 4 = 0, so, a – 4 < 0, i.e., a < 4. Now $a^2-4a<0 \Rightarrow 0<a<4$.



67. A

Sol. $R=\left(1,\frac{1+3\lambda}{1+\lambda}\right)$. It is an interior point of $y^2-4x=0$ if $\left(\frac{1+3\lambda}{1+\lambda}\right)^2-4<0$.

Therefore, $-\frac{3}{5}<\lambda<1$. But $\lambda > 0$

68. D

Sol. $\sec^2\theta+\sec\theta-(\lambda+1)=0$

$\therefore \quad \sec\theta=\frac{-1\pm\sqrt{1+4(\lambda+1)}}{2}=\frac{-1\pm\sqrt{4\lambda+5}}{2}$

For real $\sec\theta, 4\lambda+5\ge 0$, i.e., $\lambda\ge -\frac{5}{4}$

Also $\sec\theta\ge 1$ or $\sec\theta\le -1$.

$\therefore \quad \frac{-1\pm\sqrt{4\lambda+5}}{2}\ge 1$ or $\quad \frac{-1\pm\sqrt{4\lambda+5}}{2}\le -1$

$\Rightarrow \quad -1+\sqrt{4\lambda+5}\ge 2$ or $\quad -1-\sqrt{4\lambda+5}\le -2$

$\Rightarrow \quad 4\lambda+5\ge 9$ or $\quad 4\lambda+5\ge 1$

$\Rightarrow \quad \lambda\ge 1$ or $\quad \lambda\ge -1$

$\therefore \quad \lambda\ge -\frac{5}{4}$ and $\lambda\ge 1$ or $\quad \lambda\ge -\frac{5}{4}$ and $\lambda\ge -1$

$\therefore \quad \lambda\ge 1$ or $\quad \lambda\ge -1 \quad \therefore \quad \lambda\in[-1,+\infty)$

69. A

Sol. Let $P=\left(\sqrt{2}a\cos\phi, \sqrt{2}b\sin\phi\right)$. F_1 and $F_2=\left(\pm\sqrt{2}ae, 0\right)$

The area of $\Delta PFF' = \left|\frac{1}{2}\begin{vmatrix} \sqrt{2}a\cos\phi & \sqrt{2}b\sin\phi & 1 \\ \sqrt{2}ae & 0 & 1 \\ -\sqrt{2}ae & 0 & 1 \end{vmatrix}\right| = \frac{1}{2}.\sqrt{2}\sin\phi.\sqrt{2}ae$

$= 2abe\sin\phi$

$\therefore$ maximum area $= 2abc = 2ab.\frac{\sqrt{a^2-b^2}}{a}$

70. C

Sol. (α, β) is an exterior point if $16\alpha^2 + 9\beta^2 - 16\alpha - 32 > 0$.

71. B

Sol. Here, $a = \cos\alpha, b = \sin\alpha$

$b^2 = a^2\left(e^2 - 1\right) \Rightarrow \sin^2\alpha = \cos^2\alpha(e^2 - 1) \Rightarrow e^2 - 1 = \tan^2\alpha \Rightarrow e = |\sec\alpha|$.

So, ae = 1

$\therefore$ abscissae of foci $= \pm ae = \pm 1$

72. B

Sol. For the ellipse, $a^2 = 16; b^2 = a^2\left(1 - e^2\right)$

$\Rightarrow e = \frac{\sqrt{16-b^2}}{4} \Rightarrow ae = \sqrt{16-b^2}$.

For the hyperbola, $a^2 = \frac{144}{25}, b^2 = \frac{81}{25}; b^2 = a^2\left(e^2 - 1\right) \Rightarrow e = \frac{5}{4} \Rightarrow ae = 3$.

$\therefore \quad \sqrt{16-b^2} = 3$

73. B

74. C

Sol. Equation of normals are

$5x\sec\theta - 3y\,\text{cosec}\,\theta = 16$ and $y = x\tan\theta$

By eliminating θ we get $x^2 + y^2 = 64$

75. C

Sol. Any tangent to the hyperbola is $y = mx + \sqrt{a^2m^2 - b^2}$. This is the same as $y = ax + \beta$.

$\therefore \quad m = \alpha\sqrt{a^2m^2 - b^2} = \beta \Rightarrow a^2\alpha^2 - \beta^2 - b^2$

$\therefore \quad$ the locus is $a^2x^2 - y^2 = b^2$.

76. D

77. D

78. C

79. B

Sol. Clearly α, β are the roots of the equation $x^2 - 5x + 3 = 0$. Use $\alpha + \beta = 5, \alpha\beta = 3$

80. A

Sol. $x + iy = 1 - t + i\sqrt{t^2 + t + 2} \Rightarrow x = 1 - t, y = \sqrt{t^2 + t + 2}.$

Eliminating t, $y^2 = t^2 + t + 2 = (1 - x)^2 + 1 - x + 2 = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$

or $y^2 - \left(x - \frac{3}{2}\right)^2 = \frac{7}{4}$, which is a hyperbola.

81. D

82. D

Sol. Number of outcome $= \frac{{}^6C_1 \times {}^5C_1 \times {}^4C_1}{6} + \frac{{}^3C_2 \times {}^6C_1 \times {}^5C_1}{3} + {}^6C_1 = 56$

83. C

84. A

Sol. Total number of numbers without restriction = 2^5.
Tow numbers have all the digits equal. So, the required number of numbers = $2^5 - 2$.

85. A

Sol. $2\cos^2\theta + \cos\theta + (k - 1) = 0$

$\therefore \quad \cos\theta = \frac{-1 \pm \sqrt{1 - 8(k - 1)}}{4} = \frac{-1 \pm \sqrt{9 - 8k}}{4}$

For real $\cos\theta, 9 - 8k \ge 0$, i.e., $k \le \frac{9}{8}$.

Also, $-1 \le \frac{-1 \pm \sqrt{9 - 8k}}{4} \le 1$ or $-4 \le -1 \pm \sqrt{9 - 8k} \le 4$

or $-3 \le \pm\sqrt{9 - 8k} \le 5$

$\therefore \quad -3 \le -\sqrt{9 - 8k}$ and $\sqrt{9 - 8k} \le 5$

$\Rightarrow \quad 9 - 8k \le 9$ and $9 - 8k \le 25 \quad \therefore \quad k \ge 0$ and $k \ge -2$

$\Rightarrow \quad k \ge 0$

86. B

Sol. The parabolas are $y^2 - x = 0$ and $y^2 + x = 0$. The point $(\lambda, -1)$ is an exterior point if
$1 - \lambda > 0$ and $1 + \lambda > 0 \Rightarrow \lambda < 1$ and $\lambda > -1 \quad \Rightarrow -1 < \lambda < 1$

87. C

88. C

Sol. Let the GP be $a, ar, ar^2, ... (0 < r < 1)$. From the question, $\frac{a}{1 - r} = 3^3 + 3.3 - 9$

$\{\because f'(x) = 3x^2 + 3 > 0$; so, f(x) is monotonically increasing;
$\therefore$ f(3) is the greatest value in [– 2,3]}
Also, f’90) = 3, so, a – ar = 3.

Solving, a = 27(1-r) and a(1-r) = 3 we get $r = \frac{2}{3}, \frac{4}{3}$. But r < 1.

89. A

Sol. $t_n = \frac{n+2}{n(n+1)}.\left(\frac{1}{2}\right)^n = \frac{2(n+1)-n}{n(n+1)}.\left(\frac{1}{2}\right)^n = \frac{1}{n}.\left(\frac{1}{2}\right)^{n-1} - \frac{1}{n+1}.\left(\frac{1}{n}\right)^n$.

90. C

Sol. $0 \le \cos^{-1}x \le \pi$. Hence, from the question,

$\cos^{-1}\lambda = \pi, \cos^{-1}\mu = \pi, \cos^{-1}v = \pi$

$\therefore \quad \lambda = \mu = v = -1$